

Investigation of Number Sense Strategies Used By the 8th Grade Students in Turkey

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Received: April 9, 2018

Accepted: June 21, 2018

Online Published: June 24, 2018

doi:10.11114/jets.v6i7.3170

URL:<https://doi.org/10.11114/jets.v6i7.3170>

Abstract

The purpose of this research was to determine strategies used by 8th grade students in Number Sense problems. This was a case study of 28 8th grade students at three similar secondary schools in a city centre south of Turkey. A Number Sense test consisting of 25 items designed according to five main Number Sense components was used as a data collection tool. The data were analyzed using qualitative analysis methods. The students preferred Strategies Based on Rules while solving the Number Sense test compared to strategies based on number sense or Strategies Partially Based on Number Sense.

Keywords: number sense, number sense strategies, number sense component

1. Introduction

The concept of ‘Number Sense’ is significant in mathematics education. According to the National Council of Teachers of Mathematics (NCTM) standards (2000, p.32), students ought to understand numbers, ways of representing numbers, relationships between numbers and number systems, meanings of operations, relationships between numbers themselves, and do fluent computations and appropriate predictions.

There are various definitions of the concept of Number Sense in the literature. This concept and its components have been discussed by mathematics educators and cognitive psychologists and an attempt has been made to determine its components. Greeno (1991) defined Number Sense as flexible thinking, the skill of prediction in computation, and the skill of discernment about quantity. McIntosh, Reys and Reys (1992) describe Number Sense as: “a person’s general understanding of numbers and operations coupled with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and develop useful strategies for handling numbers and operations” (cited Morais and Serrazina, 2013). On the other hand, McIntosh, Reys and Reys (1992) expressed that the concept of number sense as difficult to describe.

Based on the classification developed by McIntosh, Reys, Reys, Bana and Farrell (1997), Singh (2009) used five main components to describe Number Sense: the concept of Number, using multiple expressions, equivalent expression, computing and counting strategies, and understanding effect of operation. These components explained in Table 1.

Table 1. Number sense components

Components	Explanations
Understanding the number concept	Comprehension of the value that the number represents and the size that it states (Harç 2010).
Using multiple representations	The various ways in which a number could be represented or its value recognized
Understanding the effect of the operation	An ability to perceive the value of a number during calculations and how results would be affected when operations changes (Yang, Reys and Reys 2009).
Using equivalent expressions	An ability to express a number in various forms, that is, recognition of its equivalents.
Using computing and counting strategies	Implementation of mental prediction strategies while solving problems without written computations (Kılıç, 2011)”.

In international examinations such as Trends in International Mathematics and Science Study (TIMSS) and Programme

for International Student Assessment (PISA), students are expected to interpret mathematical questions correctly, make reliable predictions and reason. When students' Number Sense develops, there is a corresponding development in skills such as reasoning, predicting, associating, mental calculation and making judgements (Harç 2010). The results of Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) examination however, show that Turkish students are not at the desired level of success. In this context, It is believed that Number Sense development is significant.

Studies have been conducted to evaluate Number Senses of primary and secondary school students (Aunio, Niemivirta, Hautamaki, Van Luit, Shi and Zhang, 2006; Markovits and Pang, 2007; Sing, 2009; Şengül and Gülbağcı, 2012), and their development of Number Sense (Alpaslan and Güner, 2013; Joram, Gabriele, Bertheau, Gelman and Subrahmanyam, 2005; Markovits and Sowder, 1994; Suh, Johnston, Jamieson and Mills, 2008). Limited information exists on the (Altay, 2010; Menon, 2004; Şengül and Gulbagci, 2012) strategies used by students while solving number sense problems in the literature. This study investigated students' usage of Number Sense strategies while solving number sense problems.

2. Method

2.1 Research Design, Setting and Participant Characteristics

A case study was conducted on 28 8th grade students, 17 girls and 11 boys, attending three similar secondary schools in a city centre in the south of Turkey.

2.2 Data Collection Tool

A number sense test (NST) consisting of some questions in the number sense scale of Singh (2009), (originally adapted from McIntosh, Reys, ReysBana, and Farrell, (1997)) was used. In the test, there were 5 components about understanding the concept of numbers, using multiple expressions of numbers, understanding effect of numbers, using equivalent expressions and using computing and counting strategies. NST consisted of 25 items about natural numbers, decimal numbers, fractions and percentages.

2.3 Data Collection and Analysis

The data were obtained through document analysis and interview techniques. The document analysis employed the use of 28 students' answer sheets. The same students were interviewed individually to find out how they thought as they solved mathematical questions listed in the Number Sense Test.

Throughout the NST implementation, students were told that they were expected to solve questions in the Number Sense test in the way that came to their mind first. After the NST, students were interviewed to determine strategies they had used while solving the questions. During the interviews, the students' answer sheets were shown to them again and they were asked to describe how they reached the solution and how they had thought through the process. Interviews were recorded using a voice recorder.

The students' answer sheets and interview responses were analyzed using qualitative analysis methods. Correct answers to each question were coded as 1 and incorrect answers coded as 0. Next, voice recordings were transcribed. The students' interviews were anonymized as S1, S2, ... S28 . Strategies used in the solving Number Sense problems were coded under four categories, i.e. Strategy(fully) Based on Number Sense (SBNS), Strategy Partially Based on Number Sense (SPBNS), Strategy Based on Rules (SBR) and the answers that have no explanation (no explanation).

3. Results and Discussion

3.1 Distribution of Scores

700 answers that was used related to the strategies ($25 \times 28 = 700$) were obtained from the students' solutions in the NST of 25 items. Additionally, the correct answers was rated as 1 in the NST of 25 items and students reached the correct answer average of $\mu = 11,46$.

3.2 Distribution of Strategies Used in Obtaining Solutions

Table 2. General distribution of the strategies that the students used in their solutions

Strategies	F	%
SBSN	160	22.85
SPBSN	17	2.42
SBR	389	55.57
No explanation	134	19.14
Total	700	100

Distribution of strategies used by students used in obtaining the 700 answers are presented in Table 2.

Majority of the answers (55,57 %) obtained from the students used Strategies Based on Rules (SBR) while solving the

problems, while 22,85 % used Strategies Based on Numbers (SBN), 19,14% had no explanation, and 2.42 % of them used Strategies Partially Based on Number Sense (SPBNS).

3.3 The 5 Components of the Numbers Sense Strategies

The strategies that students used for the problems about each number sense component were also investigated and findings approached on the basis of components.

3.3.1 Distribution of Strategies Used in Understanding the Concept of Numbers

The findings about strategies used in problems about understanding the concept of number are resented in Table 3. The majority (50,59 %) used the Strategies Based on Rules,32.14%had no explanation, 12,5 % used Strategies Based on Number Sense, and 4.76 % used the Strategies Partially Based on Number Sense.

Table 3. The distribution concerning the strategies about the component of understanding the concept of number

Strategies	F	%
SBSN	21	12.5
SPBSN	8	4.76
SBR	85	50.59
No explanation	54	32.14
Total	168	100

3.3.1.1 Interpretation of Illustrations and Interview Excerpts for NST About Understanding the Concept of Numbers

Illustration S12: A sample from students’ interviews and answer sheets from 21st item in NST is shown below (Figure 1).

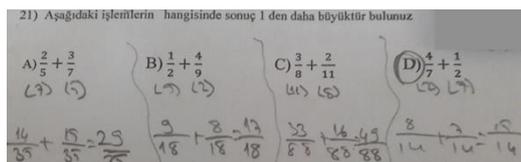


Figure1. Solution with the strategy based on rules

Interview excerpt S12: The explanation given by student was “When I equalized the denominator separately for each option, I noticed that the result in option D was bigger than 1 (S12)”.

Interpretation S12: When evaluating the solution in Figure 1 and the opinion of the student together, he/she used the strategy based on rules. The student adhered to the rules of equalizing the denominator while doing addition in rational numbers.

Interview excerpt S3: Another student, S3, explained his/her solution to the same item as “Firstly, both fractions in option A are smaller than half so they are smaller than 1. In the option B, there is a half fraction and a fraction smaller than half. In the option C, both fractions are smaller than half. In the option D, there is a fraction bigger than half and a fraction that is the sum of half so it represents a number bigger than 1 (S3).”

Interpretation S3: Thus, it can be said that S3 used the strategy based on number sense.

3.3.2 Distribution of Strategies Used in the Component of Multiple Representations

The findings about strategies used in the problems regarding the component of using the multiple representations are presented in Table 4. that the majority (34.28%) of the answers given to the items containing the component of using multiple representations had no explanation, 33,57 % used the Strategies Based on Number Sense, 3,57 % of them used Strategies Partially Based on Number Sense and 28,57 % of them used the Strategies Based on Rules.

Table 4. The distribution concerning the strategies about the component of using the multiple representations

Strategies	F	%
SBSN	47	33.57
SPBNS	5	3.57
SBR	40	28.57
No explanation	48	34.28
Total	140	100

3.3.3 Distribution of Strategies Used in the Component of Understanding the Effect of Operations

The findings regarding strategies used in problems about the component of understanding the effect of the operation are

presented in Table 5. The majority (66,43 %) used Strategies Based on Rules, 23,57 % used Strategies Based on Number Sense, 10% had no explanation and 0% used Strategies Partially Based on Number Sense.

Table 5. The distribution concerning the strategies about the component of understanding the effect of the operation

Strategies	F	%
SBSN	33	23.57
SPBNS	0	0
SBR	93	66.43
No explanation	14	10
Total	140	100

3.3.3.1 Interpretation of Illustrations and Interview Excerpts for NST Question in the Component of Understating the Effect of the Operation

Illustration S9: A sample from students' interviews and answer sheets from the NST about items that were involved in the component of understanding the effect of the operation are given below (Figure 2).

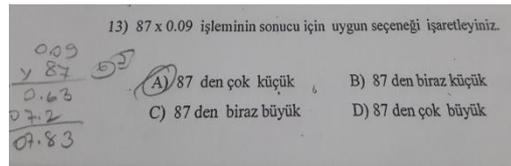


Figure 2. Solution with the strategy based on rules

Interview excerpt S9: The explanation of the student who gave the answer in Figure 2 for the 13th item in NST is as follows “When I multiplied 89 with 0,09, I got 7,83 and as this value was much smaller than 89, the correct answer was the option A.”

Interpretation S9: When evaluating the solution in Figure 2 and the opinions of the students the student used the strategy based on rules.

Interview excerpt S12: The explanation of S12 about the solution of this item was “When we multiply 89 with 1, we get 89. When we multiply 89 with 0,5, that is half, we get the half of 89, that is a smaller result. 0,09 is much smaller than 1 so the answer will be far smaller than 89 (S12).”

Interpretation S12: The student used the strategy based on number sense.

3.3.4 Distribution of Strategies Used in the Component of Using Equivalent Expressions

The findings about strategies were used in the problems regarding the component of using equivalent expressions are presented in Table 6. The majority (76,78 %) used Strategies Based on Rules, 18,75 % used Strategies Based on Number Sense, 2.67% had no explanation and 1.78% used and Strategies Partially Based on Number Sense.

Table 6. The distribution concerning the strategies about using the equivalent expressions

Strategies	F	%
SBSN	21	18,75
SPBNS	2	1,78
SBR	86	76,78
No explanation	3	2,67
Total	112	100

3.3.5 Distribution of Strategies Used in the Component of Using Computing and Counting

The findings about strategies that were used in problems regarding the component of using strategies of computing and counting are presented in Table 7. The majority (60,71 %) used Strategies Based on Rules, 26,42 % used the Strategies Based on Number Sense and., 11.42% had no explanation and 1.42% used Strategies Partially Based on Number Sense.

Table 7. The distribution concerning the strategies about the component of using computing and counting strategies

Strategies	F	%
SBSN	37	26.42
SPBNS	2	1.42
SBR	85	60.71
No	16	11.42
Total	140	100

3.3.5.1 Interpretation of Illustrations and interview Excerpts for NST Question in the Component Using Computing and Counting Strategies

Illustration S27: A sample from the students' interviews and answer sheets about the items were involved in the component of using computing and counting strategies are given below (Figure 3).

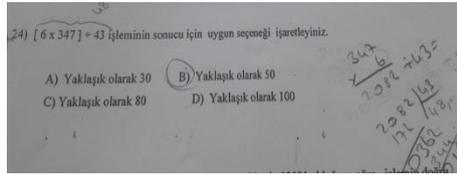


Figure 3. Solution with the strategy based on rules

Interview excerpt S27: The explanation of the student who gave the answer in Figure 3 for the 24th item in NST is as follows “First, I multiplied 347 with 6, I got 2082. Then, I divided the result by 43 and I got 48,4. So the correct answer is the option B (S27).”

Interpretation S27: When evaluating the solution in Figure 3 and the opinions of the student, the student used the strategy based on rules.

Interview excerpt S10: The explanation of S10 about the solution of this item was “I rounded 347 up to 350. I thought 43 as 40 and I made mental calculus. I predicted 50 approximately as a result of the division of 2100 by 40 (S10).”

Interpretation S10: It can be interpreted that S10 used the strategy based on number sense

4. Conclusion and Recommendations

The students used the Strategies Based on Rules in computing their solutions more. We recommend the usage of the Strategies of Number Sense be presented during their education. Additionally, the component of understanding the concept of number employed Strategies Based on Number Sense the least. Also, the component of using Multiple representations did not have Strategies Based on Rules as the highest Were commend the usage of the strategies of number sense can be presented during their education.

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