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Prospective Middle School Mathematics Teachers' (PMTs) Content Knowledge about Concepts 'Fraction' and 'Rational Number'

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Abstract

The present study aims to examine the content knowledge of prospective middle school mathematics teachers (PMTs) on the concepts 'fraction' and 'rational number' and relationship between them. This study, in which mixed type (quantitative+qualitative) research approach is used, was carried out with 164 PMTs in total, who study in the department of middle school mathematics teaching of the Faculties of Education in three different state universities in Turkey. "Fraction-Rational Number Test (FRNT)" consisting of six questions was used as data collection tool. Descriptive statistics were used in the analysis of the data, and each question was separately assessed. Results showed that approximately 37% of the PMTs answered the questions in FRNT as completely true, and approximately 23% of them answered as wrong category. It was also found out that some of PMTs confused the concepts of fraction and rational number and used them interchangeably. From these results, it can be said that the content knowledge of PMTs on 'fraction' and 'rational number' concepts and relationship between them is not at sufficient level.

Keywords: fraction, rational number, content knowledge, prospective middle school mathematics teachers (PMTs)

1. Introduction

Fractions and rational numbers are two crucial topics on which there is abundant research and discussion in the field of mathematics education. Both in past and recent times, many mathematics educators (e.g., Behr, Lesh, Post and Silver, 1983; Bezuk and Bieck, 1993; Erdem, 2015; Erdem, Gökkurt, Şahin, Başıbüyük and Soylu, 2015; Kieren, 1993; Lamon, 2007; Lesh, Behr, & Post, 1987; Ma, 1999; Mack, 1995; Moss and Case, 1999; Ni, 2001; Ni & Zhou, 2005; Novillis, 1976; Stafylidou and Vosniadou, 2004; Tirosh, 2000; Ünlü and Ertekin, 2012) agree that fractions and rational numbers have notoriety for the difficulty that school students experience in learning them. Many reasons that are effective in experiencing this difficulty are given in the following literature:

- Fractions and rational numbers are presented predominantly by means of rules rather than conceptual instruction (Aksu, 1997; Bezuk & Bieck, 1993; de Castro, 2008; Gökkurt, Şahin, Soylu, & Soylu, 2013; Moss & Case, 1999; Tirosh, 2000; Toluk-Uçar, 2009)
- Fractions and their properties do not exhibit much accord with natural numbers and their properties with which the students are familiar (Stafylidou & Vosniadou, 2004; Tirosh, 2000)
- The numerator and denominator of the fraction/rational number are perceived as two unrelated whole numbers (Doğan & Yeniterzi, 2011; Şiap & Duru, 2004)
- Teachers or prospective teachers have had incomplete or memorized knowledge about fractions in their previous experiences (Toluk-Uçar, 2009)
- An expression like $\frac{a}{b}$ can have different meanings. These meanings are explained in the literature as follows: "a) *part-whole comparison* - it signifies the relationship between a whole and a part, b) *ratio* - it signifies the relationship between two quantities, c) *quotient* - it signifies the division operation, d) *operator* - it signifies a function that transforms a set into another set with a/b times as many elements, e) *measurement* - it signifies the number of units or subunits that "equal" the object measured" (Behr et al., 1983).

It is thought that especially two factors: (1) there is a little difference between fraction and rational number, (2) both of them are written in the form $\frac{a}{b}$ can be effective in having difficulty in comprehending the relationship between fraction and rational number.

1.1 Theoretical Framework on Relationship between Fraction and Rational Number

"Fractions" and "rational numbers" are two genuinely associated, but nonexchangeable, terms from both a mathematical and a psychological point of view (Ni and Zhou, 2005). Piaget, Inhelder, and Szeminska (1960) identify seven sub concepts for a conceptual analysis of fraction: (a) the whole is divisible, potentially composed of separable elements; (b) a fraction implies a determinate number of parts; (c) the sub division must be exhaustive; (d) there is a fixed relation between the number of parts and the number of divisions; (e) the parts have a nesting or hierarchical character; (f) the whole is conserved under sub division; and (g) all parts must be equal (cited in Hiebert & Tonnessen, 1978). Fraction is shortly defined as one or several equal parts of a whole (Baykul, 2005). Rational number is usually described as a number that is or can be expressed in the form $\frac{a}{b}$, where a and b are integers, and b is non-zero (Vamvakoussi and Vosniadou, 2010). As fractions indicate amount and the value of even a very small part of this amount that will be higher than 0 (zero), fractions cannot have negative values. Otherwise, students can consider any division in the form $\frac{a}{b}$ as a fraction.

Fractions can sometimes be confused with rational numbers and perceived as a number set. Indeed, Usiskin (1979) remarked that fractions are not components of a number system. Lamon (2007, p. 635) describes the relationship between fraction and rational number by explaining "$\frac{2}{3}$, $\frac{6}{9}$ and $\frac{10}{15}$, … these fractions are different numerals designating the same rational number. A single rational number underlies all of the equivalent forms of a fraction". In the broadest sense of the term, fractions are the expressions that are used to represent rational numbers and that can exist in infinite numbers. Behr, Harel, Post and Lesh (1992) explained the relation between rational numbers and fractions: "Rational numbers are elements of an infinite quotient field consisting of infinite equivalence classes, and the elements of the equivalence classes are fractions". Ni (2001) reported in a similar way, explaining that every fraction belongs to a single equivalence class generated by a single multiplicative equation, and each equivalence class defines a distinct rational number. Equivalence of fractions and its relation to rational numbers was clearly explained by Vamvakoussi and Vosniadou (2010) as follows: "One could say that 0.5, 500/1000 and also 1/2 or 7/14, are all rational numbers. However, a closer examination shows that they all have the same value, thus the accurate thing to say would be that these are alternative representations of the same rational number. A mathematician would define this rational number as the equivalence class of pairs [a,b] such that 2a = b, and would say that all the above are representatives of this class".

Hilbert (2011) reported that the set of all rational numbers are the fractions such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$,… $\frac{3}{7}$,… Balcı (2006) stated that p and q in $\mathbb{Q}=\{\frac{p}{q} : p, q \in Z\ ve\ q \neq 0\}$ set must be relatively prime, otherwise, there will be an infinite number of elements that are equal in $\mathbb{Q}$ set. King (2010) stated that rational numbers are common fractions of which numerators and denumerators (the denumerator cannot be zero) are whole numbers. That the numerator and denumerator must be relatively prime is also revealed by (Papp, 1994) when proving that $\sqrt{2}$ is not rational.

On the other hand, considering that different elements in a number set represent different points on a number line, it can be understood that in rational numbers the numerator and denumerator must be relatively prime. As seen in Figure 1, $\frac{1}{2}$, $\frac{3}{6}$ and $\frac{6}{12}$ are fractions that represent same point on the number line.

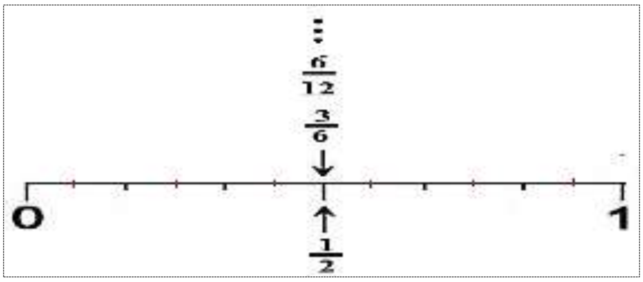

Figure 1. Equivalent fractions showing the same point on the number line

Stafylidou and Vosniadou (2004) state a key knowledge that fractions do not have unique successors. It can be said that only $\frac{1}{2}$ is a rational number among the fractions above. In other words, A cannot indicate a set of number as the elements in the structure A= $\{\frac{1}{2}, \frac{3}{6}, \frac{6}{12}, \ldots\}$ indicate the same point on the number line. Wong and Evans (2007) showed that each fraction in the set is interchangeable with the others and defined this knowledge as fraction equivalence.

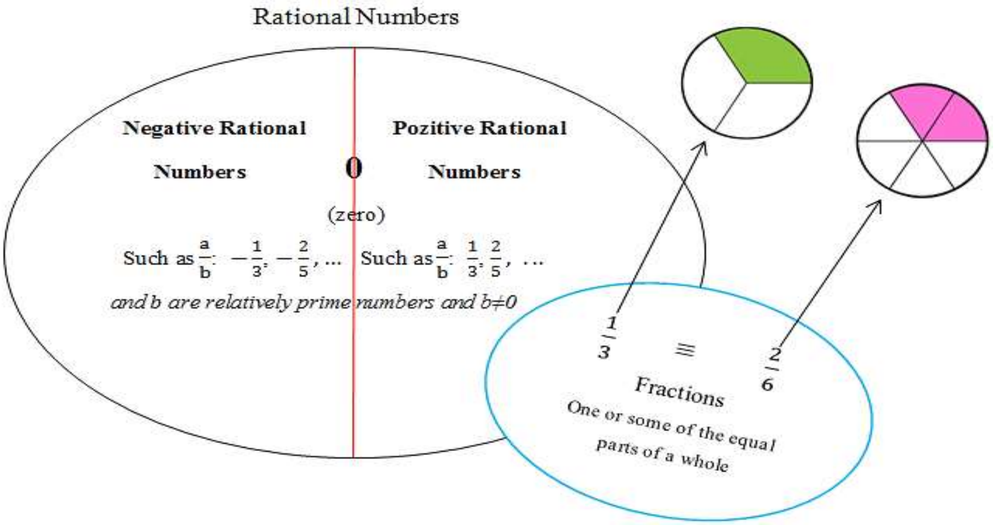

Figure 2. Explanatory framework for the relationship between fraction and rational number

From all these researches, it is possible to explain the relationship between the fraction and rational number as follows: $\frac{1}{2}$ can be taken as both a rational number and a fraction. However, expressions such as $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$,…, which are equivalent to this fraction, are fractions, but they are not rational numbers since they do not fit the term rational number (*the fact that the numerator and denominator must be relatively prime*). It can be said that each positive rational number is also a fraction (See Figure 2). This provides us with the conclusion that a rational number can be represented by more than one fraction. The difference between fraction and rational number can be explained in another way that each rational number represents an equivalence class and fractions generated by multiplying both numerator and denominator with the same positive whole number are the elements of these equivalence classes. Accordingly, it is remarked that fractions cannot be a number set since this contradicts with the fact that the set is well defined (Erdem et al., 2015).

Then, it can be asked that what kind of expression $-\frac{2}{3}$ is? It is obvious that it cannot be a fraction as this expression is negative. This expression is a rational number as the numerator and denumerator are relatively prime. As for the expression $-\frac{2}{4}$, it also cannot express a fraction, and it cannot be a rational number as the numerator and denumerator are not relatively prime (the common divisor of the numerator and denumerator is 2). Then, $-\frac{2}{4}$ can be accounted as an expression that can be turned into a rational number by abbreviation. It is possible to explain this as follows with another example: For example, what kind of an expression is $\frac{\frac{1}{2}}{\frac{3}{4}}$? Here, it does not indicate a rational number as the expressions in the numerator and denumerator are not whole numbers. We can express this expression as a rationalizible mathematical expression (*by multiplying the numerator and denumerator with the multiplicative inverse of the rational number in the denumerator*).

The place of the teacher, who is the main driver of the process in learning environment, is very important in teaching mathematics. In the literature, it is indicated that the teacher's competence has a crucial place in teaching mathematics (Erdem et al., 2015; Erdem and Soylu, 2013; Gürbüz, Erdem and Gülburnu, 2013; NCTM, 2000; Romberg and Carpenter, 1986). Within the scope of teacher's competence, content knowledge and pedagogical content knowledge are the most important factors in ensuring that the process of instruction is effective. In other words, in order to teach effectively, a teacher must possess adequate content knowledge and is capable of effectively transmitting this knowledge to the students. Literature also emphasized that the content knowledge and pedagogical content knowledge of the teacher are essential for an effective teaching (Ball, 1988; 1990; Cankoy, 2010; Davis and Simmt, 2006; Erdem, 2015; Erdem et al., 2015; Erdem and Soylu, 2013; Gökkurt et al., 2013; Gürbüz et al. 2013; Hill, Rowan and Ball, 2005; Rowan, Chiang, & Miller, 1997; Shulman, 1986; 1987; Şahin, Gökkurt, Başıbüyük, Erdem, Nergiz and Soylu, 2013; Tchoshanov, 2011). That prospective teachers who will become the future teachers can be successful in their career depends on education they receive during their university years and how capable they are when they graduate. Indeed, it is possible to see studies showing that university education plays important role in managing the teaching process well (Arslan and Özpınar, 2008; Erdem, 2015; Erdem et al., 2015; Erdem and Soylu, 2013; Gürbüz et al., 2013; Hill et al., 2005; Peker, 2009; Smith, 2000).

1.2 Background Information: Middle School Mathematics Teaching Curriculum in Turkey

In Turkey, the educational context is structured into five levels: preschool education (aged 3–6), primary schools (Grades 1–4, aged 6-9), middle schools (Grades 5–8, aged 10–13), high schools (Grades 9–12, aged 14-17), and universities. All schools throughout the country are supposed to use the same curricula, which was developed and implemented by the Ministry of National Education. In the middle school mathematics curriculum, there are five learning domains, which are *numbers and operations, algebra, geometry and measurement, data analysis*, and *probability.* "Numbers and Operations" has the highest number of course hours in comparison with others and fractions and rational number are the two broad topics of this learning domain (Ministry of National Education [MNE] (2013). While the 'fractions' topic is taught in 5th and 6th grades, 'rational numbers' is the topic taught only in 7th grade. While there is an emphasis on the meaning of fraction concept and addition and subtraction with fractions in 5th grade, multiplication and division operations with fractions are taught in 6th grade and the emphasis is shifted to the meaning of rational numbers and addition, subtraction, multiplication and division in rational numbers in 7th grade.

Middle school mathematics teaching program consists of 4-year university level education. Pure math courses such as General Mathematics, Abstract Mathematics courses, other education courses are given in the first two years and in the third year, with ongoing pure math courses some content and pedagogical content courses such as Special Teaching Methods-I and II courses are given. Special Teaching Methods courses aim to teach conceptual issues related to topics (fractions and rational numbers etc.) in middle school math teaching curriculum, their usages in daily life, misconceptions students have and how they can handle them and various teaching methods. In the final year, together with some general education courses, they complete their math teaching practices in middle schools. Also, students are able to take some elective courses related to field education except for final year. A teacher candidate who succesfully complete this education program can start teaching in 5th through 8th grades provided that they get necessary grades form a central exam they need to take.

One reason why fraction and rational number concepts were chosen in the current study is that, as mentioned in the literature above, students find it difficult to understand these concepts, another reason is emphasis on these two concepts in middle school math teaching curriculum. The reason why teacher candidates were chosen as study group is to detect their content knowledge and help them make necessary corrections as they will be teachers of these topics in their future career. The current study contributes to existing literature on fractions and rational numbers since it tries to reveal PMTs' content knowledge about 'fraction' and 'rational number' concepts and the relationship between them. In this context, the objectives of the study were: (1) to determine the content knowledge level of PMTs on concepts of 'fraction' and 'rational number' (2) to examine their content knowledge on the relationship between these concepts.

2. Method

2.1 Research Design

The research is a descriptive study that was carried out in accordance with the mixed (qualitative+quantitative) research approach. In this approach, qualitative and quantitative data exist together in order to ensure data variation (McMillan and Schumacher, 2010).

2.2 Participants

The participants consist of 164 prospective middle school mathematics teachers (PMTs) studying in the Middle School Mathematics Teaching program of the Faculties of Education of three state universities in Turkey. The participants were determined from different universities, and it was tried to establish whether there are mutual cases between varied situations. The reason why only PMTs who are at the last grade of their education in university were included in the study is that they received *Special Teaching Methods I-II* courses, which are important in terms of their having content knowledge and pedagogical content knowledge. The participants were given codes as S_1, S_2, S_3 in order to keep their identities private.

2.3 Instrumentation

"Fraction-Rational Number Test (FRNT)" was used as the data collection tool. In the first two questions in FRNT, the definitions of the fractions and rational numbers, and whether the given expressions are fraction or rational number were asked in the remaining four questions. Informal interviews held from time to time with candidate teachers before the start of the research process were used in determination of the questions in the test. As a result of these interviews, the statements that candidate teachers had difficulty in understanding were detected. The opinions of three faculty members who are specialized in teaching mathematics were taken for the validity of the FRNT. With the help of pilot study conducted with the participation of 20 candidate teachers, it was decided that 30 minutes should be given to candidate teachers for FRNT in the real application and the expressions that were hard to understand or led to misunderstandings were corrected. The FRNT was put into its final form by making the necessary corrections in line with the expert opinions. Also, Cronbach Alfa coefficient was calculated as ".852". Furthermore, candidate teachers were asked to write their explanations on the questions in a detailed way in order to obtain a healthier and clearer data.

2.4 Data analysis

Descriptive statistics were used in the analysis of the data and each question was handled separately. The scoring rubric developed by Erdem (2011) was used in the scoring of the questions in FRNT (See Table 1). The frequency and percentage tables of each question in the test are shown considering the levels in the rubric. Answers given to the questions were scored independently by two mathematics teaching experts, and it was tried to calculate the scoring reliability. The consistency between the scoring was determined as 87% ($p=.000$, $r=.868$). In addition, some answers of each question were directly presented as it is believed that quotations would be effective in plainly reflecting individuals' thoughts (Yin, 2011).

Table 1. Scoring rubric

Level	Score	Explanation
Completely true	5	Expressions that are deemed completely true
Partially true-A	4	Missing expressions despite a completely true answer
Partially true -B	3	Partially true answers given in reference to a correct reason
Partially true -C	2	Expressions that can be deemed partially true in reference to an incorrect reason or without giving a reason
Wrong	1	Expressions that are completely incorrect or that have to complete relationship with the question
Blank	0	Expressions that are left blank or in which the question itself is written as the answer

3. Results

This section includes the descriptive statistics results of the answers given by PMTs in FRNT, and some participant answers about each question are presented separately.

Table 2. Descriptive statistics results of the answers given to questions in FRNT

Question	Category											
	Completely True		Partially True-A		Partially True -B		Partially True -C		Wrong		Blank	
	f	%	f	%	f	%	f	%	f	%	f	%
1	48	29.3	44	26.8	18	11.0	12	7.3	41	25.0	1	0.6
2	34	20.7	6	3.7	92	56.1	28	17.1	2	1.2	2	1.2
3	92	56.1	14	8.5	46	28.0	7	4.3	4	2.4	1	0.6
4	114	69.5	8	4.9	8	4.9	13	7.9	20	12.2	1	0.6
5	63	38.4	3	1.8	68	41.5	3	1.8	25	15.2	2	1.2
6	12	7.3	1	0.6	0	0	9	5.5	137	83.5	5	3.0
Mean	61	36.9	13	7.7	39	23.6	12	7.3	38	23.3	2	1.2

As is seen in Table 2, approximately 37% of candidate teachers (*N=61, 36.9%*) answered the questions in FRNT in the completely true category. It was determined that participants correctly answered most in question 4 and least in question 6. On the other hand, approximately 23% of the participants (*N=38, 23.3%*) answered the questions in FRNT in the *wrong* category. As is seen in Table 2, the participants wrongly answered most in question 6 and least in question 2.

Question 1: *What is fraction? Please explain.*

In this question, candidate teachers should use the expression *"one or some of the equal parts of a whole"* or expressions that bear a similar meaning with this expression for the concept 'fraction'. It was found out that 29.3% of the participants answered this question in the *completely true* category, 26.8% *partially true-A*, 11.0% *partially true-B*, 7.3% *partially true-C*, 25.0% *wrong* and 0.6% gave answers in the *blank* category. It was concluded that, when compared to all questions, this question is the most answered question in partially true-A category, and one of the least answered questions in *blank* category. Some participant answers regarding this question are as follows.

- *To divide a whole into equal parts (S1).*
- *Fraction is one or some parts of a whole (S33).*
- *The concept used in order to express one or some parts of a whole that is divided into equal parts (S40).*
- *Fraction is all of the equal parts of a whole. Each number is actually a fraction (S130).*
- *The ratio of the part to the whole (S148).*
- *Expressions defined as* $\{ \frac{a}{b} : a, b \in Z+ \text{ and } b \neq 0\}$ *are fractions (S157).*

Question 2: *What is Rational Number? Please explain.*

In this question, candidate teachers should use the expression $\{ \frac{a}{b} : a, b \in Z, (a,b) = 1 \text{ and } b \neq 0\}$ or a similar expression for the concept 'rational number'. When the results were examined, it was detected that 20.7% of the participants answered in the *completely true* category, 3.7% in *partially true-A*, 56.1% in *partially true-B*, 17.1% in *partially true-C*, 1.2% in *wrong* and 1.2% answered in the *blank* category. When compared to all questions, it was determined that this question is the most answered question in *partially true-B* category, and the least answered question in *wrong* category. Some participant answers with regard to this question are as follows.

- *Rational number is different than the concept fraction. They can be confused as they have many things in common. A rational number can be written as* $\frac{a}{b}$ *and a and b must be relatively prime. So, each rational number is a fraction, but not all fractions are rational numbers (S14).*
- *Fractions of which numerators and denumerators are relatively prime (S18).*
- *Numbers of which numerator and denumerator are relatively prime. They can be positive or negative (S38).*

- *The simplest number that can be written as* $\frac{a}{b}$ *(b ≠0) (S85).*
- *The set comprising whole numbers, natural numbers and cardinal numbers is named as the set of rational numbers. And the elements of this set are called rational numbers (S87).*
- *Expressions defined as* $\{\frac{a}{b} : a, b \in Z, (a, b) = 1 \text{ ve } b \neq 0\}$ *are rational numbers (S157).*

Question 3: *Is the expression* $\frac{1}{3}$ *a fraction or a rational number? Please explain.*

In this question, candidate teachers should use the expression "*a rational number as the numerator and denumerator are relatively prime, and also a fraction as it represents one of the 3 equal parts of a whole*" or a similar expression for $\frac{1}{3}$. As a result of the analyses, it was found out that 56.1% of the participants answered this question *completely true*, 8.5% as *partially true-A*, 28.0% as *partially true-B*, 4.3% as *partially true-C*, 2.4% as *wrong* and 0.6% as *blank*. When compared to all of the questions, it was found out that this question is one of the least answered question in the *blank* category. Some participant answers with regard to this question are as follows.

- $\frac{1}{3}$ *is a fraction. A whole was divided into 3 equal parts, and one of the parts is taken. It is at the same time a rational number. For, the numerator and denumerator are relatively prime (S1).*
- *Both. It is a fraction as its numerator and denumerator are (+) and a rational number as they are relatively prime (S19).*
- *It is a fraction. 1 of the 3 equal parts are taken (S55).*
- *(3,1)=1. And also, a whole is divided into 3 equal parts and one of the parts is taken. Then, it is both a fraction and a rational number (S132).*
- *It is both a rational number and a fraction. Each rational number is a fraction (S129).*
- *It can be both fraction and rational number. It is read as 1 in 3 if it is a fraction (S142).*

Question 4: *Is the expression* $-\frac{2}{5}$ *a fraction or a rational number? Please explain.*

In this question, candidate teachers should use the expression "*it is a rational number as the numerator and denumerator are relatively prime, but it is not a fraction as fractions cannot have negative values*" or a similar expression for $-\frac{2}{5}$. As a result of the analyses, it was determined that 69.5% of the participants answered this question as *completely true*, 4.9% as *partially true-A*, 4.9% as *partially true-B*, 7.9% as *partially true-C*, 12.2% as *wrong* and 0.6% as *blank*. When compared to all questions, it was concluded that it is the most answered question in the *completely true* category and one of the least answered questions in the *blank* category. Some participant answers to this question are as follows.

- *Fractions cannot be negative, thus it is only a rational number (S10).*
- *It is a rational number. For, we cannot establish a part-whole relationship in negative expressions (S17).*
- *It is both a fraction and a rational number. That it is negative does not matter (S95).*
- $-\frac{2}{5}$ *expression is a negative fraction (S117).*
- *Negative numbers are rational numbers. Negative numbers cannot be fractions. For negative numbers cannot be taken from a whole (S154).*
- *It is a rational number. It cannot be expressed in fraction. We cannot take -2 of 5 parts (S159).*

Question 5: *Is the expression* $\frac{2}{4}$ *a fraction or a rational number? Please explain.*

In this question, candidate teachers should use the expression "*it is not a rational number as the numerator and denumerator are not relatively prime; and it is a fraction as it expresses 2 parts of a whole that is divided into 4 parts*" or a similar expression bearing the same meaning for $\frac{2}{4}$. As a result of the analyses, it was determined that 38.4% of the participants answered this question as *completely true*, 1.8% as *partially true-A*, 41.5% as *partially true-B*, 1.8% as *partially true-C*, 15.2% as *wrong* and 1.2% as *blank*. When compared to all questions, it was found out that it is the least answered questions in *partially true-C* category. Some participant answers to this question are as follows.

- *It is a fraction, but not a rational number. As 2 and 4 are not relatively prime (S2).*
- *It is divided into 4 parts and 2 parts are taken (S63).*
- $\frac{2}{4} = \frac{1}{2}$ *means half. It is taught at primary school level. Because it is positive. It is taught as a rational number at secondary school level. Rational numbers include fractions (S100).*
- *It is both a fraction and rational number.* $2,4 \in Z+$ *and* $4 \neq 0$ *(S150).*
- *It is a fraction. It is not a rational number as* $(2,4)=2 \neq 1$ *(S161).*
- *2 and 4 are not relatively prime. Thus, it is a fraction (S163).*

Question 6: *Is the expression* $-\frac{3}{6}$ *a fraction or a rational number? Please explain.*

In this question, candidate teachers should use the expression *"It is not a fraction as it is a negative expression, and not a rational number as 3 and 6 are not relatively prime. It is a mathematical structure that can be turned into a rational number by abbreviation"* or a similar expression bearing the same meaning for $-\frac{3}{6}$. As a result of the analyses, it was determined that 7.3% of the participants answered this question as *completely true*, 0.6% as *partially true-A*, 0.0% as *partially true-B*, 5.5% as *partially true-C*, 83.5% as *wrong* and 3.0% as *blank*. When compared to all questions, it was concluded that question is the most answered question in the *wrong* category and not answered at all in *partially true-B* category. Some participant answers to this question are as follows.

- *It is not a rational number as 3 and 6 are not relatively prime. It is not a fraction either, as (-) cannot be fraction (S3).*
- *It is not a fraction. Not a rational number either. It would be a rational number if it was* $-\frac{1}{2}$ *(S13).*
- *This expression is neither a rational number nor a fraction. It cannot be a fraction as it is negative; it is not a rational number too as 3 and 6 are not relatively prime. This is a division (S14).*
- *It is a rational number, because only fractions are positive (S80).*
- *It is a negative fraction as 3 and 6 are not relatively prime (S137).*
- *It is neither a fraction nor a rational number. It is the expanded form of* $-\frac{1}{2}$ *with 3 (S160).*

In summary, it was concluded in this study that (a) approximately 37% of the participants answer the questions in FRNT completely true and approximately 23% as wrong, (b) they answered as completely true most in the question on whether the expression $-\frac{2}{5}$ is a fraction or a rational number, (c) they answered as wrong most in the question on whether the expression $-\frac{3}{6}$ is a fraction or a rational number and (d) they answered the question on the definition of rational number least wrong. It can be said that the content knowledge of PMTs on 'fraction' and 'rational number' concepts is not at sufficient level. It was seen that PMTs did not comprehend relationship between fraction and rational number and used them interchangeably. On the other hand, the most striking deficiency experienced by PMTs is that they did not realize that numerator and denumerator in rational numbers to be relatively prime. The most correct knowledge, arised from PMTs' responses, is that (1) fraction is the equal parts of a whole and (2) each positive rational number is also a fraction.

4. Discussion and Conclusions

In this research, the content knowledge of PMTs on the concepts of fraction and rational number, and the relationship between these concepts were examined. As a result of the analysis, it was found out that less than half of the candidate teachers (37%) in average answered the questions in the *completely true* category. This result shows that the content knowledge of PMTs on the concepts fraction and rational number is not at a sufficient level. From this result, it can be said that lessons such as Special Teaching Methods I-II received during university years that contribute to the content knowledge and pedagogical content knowledge are not sufficient in creating the content knowledge of candidate teachers on the concepts fraction and rational number. Indeed, the university education received before practicing the teaching profession is quite important in terms of obtaining the content knowledge and knowledge on teaching the content by the candidate teacher, and this is also emphasized in the relevant literature (Arslan and Özpınar, 2008; Erdem, 2015; Erdem et al., 2015; Erdem and Soylu, 2013; Gürbüz et al., 2013; Hill et al., 2005; Peker, 2009; Smith, 2000). Considering the findings of the study, it was determined that the percentage of the participants who answer the question on the definition of the concept fraction as completely true is 29.3%. That the percentage of the participants answering in the completely true category is low and this is one of the least answered questiones in the blank category show that the content knowledge of candidate teachers on the concept fraction is defective. In parallel, it is observed that the content knowledge and pedagogical content knowledge of mathematics teachers and candidate mathematics teachers is not at a sufficient level (Ball, 1990; Gökkurt et al., 2013; Işıksal, 2006; Li and Kulm, 2008; Ma, 1999; Newton, 2008; Simon, 1993; Ünlü and Ertekin, 2012). Upon examining the written expressions of the participants, it was determined that they gave wrong answers such as fractions can take negative values and each fraction is a rational number. On the other hand, it was seen that some candidate teachers gave answers using the mathematical language such as $\{\frac{a}{b} : a, b \in Z^+ and\ b \neq 0\}$ for the concept fraction. That candidate teachers using this expression for the fraction indicated that $a, b \in Z^+$ can be interpreted that they know that fractional expressions must be positive. Furthermore, it was seen that some of the participants know that in fractional expressions, it is not an obligation for the numerator and denumerator to be relatively prime. In other words, it was identified that some candidate teachers know that both dividing a whole into two equal parts and taking one part of it ($\frac{1}{2}$), and dividing it into four equal parts and taking two equal parts ($\frac{2}{4}$) mean fraction.

It was concluded that the percentage of the participants who answered the question on the definition of rational number as completely true is 20.7%. This result shows that the content knowledge of candidate teachers on the concept rational number is also defective. Upon examining participant answers, wrong answers such as it is not an obligation for the numerator and denumerator in rational numbers to be relatively prime, and a rational number is the same thing as a fraction were encountered. On the other hand, correct answers were such as the numerator and denumerator in rational numbers must be relatively prime, *(the simplest form of a number that can be written as* $\frac{a}{b}$ *(b ≠0)), rational numbers can take negative values* ($\{\frac{a}{b} : a, b \in Z, (a, b) = 1\ and\ b \neq 0\}$) [$(a, b) = 1$ *means that a and b are relatively prime*]. In the question on whether the expression $\frac{1}{3}$ is a fraction or a rational number, it was determined that the percentage of the participants that answered completely true is 56.1%. It was also determined that this question is one of the least answered questions in *blank* category, when compared to all questions. This result shows that the content knowledge of candidate teachers with regard to the expression $\frac{1}{3}$ is better. It can be said that the expression $\frac{1}{3}$ indicates both a fraction and a rational number have been effective in this result. In other words, $\frac{1}{3}$ can be expressed as a part of a whole that is divided into three equal parts, and at the same time a rational number as the numerator and denumerator are relatively prime. As a consequence, a small number of participants (2.4%) answered this question as wrong. The percentage of the participants answering the question on whether the expression $\frac{2}{4}$ is a fraction or a rational number as completely true was determined to be 38%. It was also determined that this question was the least answered question in *partially true-C* category when compared to all of the questions. It can be said that the expression $\frac{2}{4}$ is positive and thus, can be easily regarded as a fraction has been effective in this result. Participants gave answers indicating that $\frac{2}{4}$ is a rational number as *"It is both a fraction and a rational number. 2, 4* $\in Z^{+}$ *and* $4 \neq 0$" although the numerator and denumerator are not relatively prime.

In the question on whether the expression $-\frac{2}{5}$ is a fraction or a rational number, it was found out that the percentage of the participants who answered completely true is 69.5%. It was determined that this question is the most answered question in the *completely true* category and one of the least answered questions in *blank* category. It can be said that the expression $-\frac{2}{5}$ cannot be a fraction as it is negative and thus is a rational number has been effective in being a rational number. It can also be said that different results would be obtained if the numerator and denumerator were not relatively prime. Indeed, it was identified that the percentage of the participants who answered the question on whether the expression $-\frac{3}{6}$ is a fraction or a ration number, in which only difference between $-\frac{2}{5}$ is that in the first expression the numerator and denumerator are not relatively prime, decreased to 7.3%. In this question, that the expression $-\frac{3}{6}$ is negative strengthens the notion that it cannot be a fraction. It also cannot be expressed as a rational number as the numerator and denumerator are not relatively prime. It can be said that this uncertainty led to the low level of correct answers by candidate teachers. Indeed, $-\frac{3}{6}$ cannot be a fraction as it is a negative expression; and cannot be rational number as the numerator and denumerator are not relatively prime. That candidate teachers are unable to understand this question is also understood from a participant answer as "*It is not a rational number; it is not a fraction at all. I don't know what to say.*" Then what kind of an expression is $-\frac{3}{6}$? It is an expression that can be turned into a rational number by abbreviation. Similarly, a candidate teacher explained this question as "*This expression is neither a rational number nor a fraction. It cannot be a fraction as it is negative; it is not a rational number too as 3 and 6 are not relatively prime. This is a division.*"

When the result that only 37% of the participants answered questions in FRNT as completely true and collectively assessing the explanations in the answers are considered, it was concluded that the concepts fraction and rational number are confused and used interchangeable. Two main differences that must be known about these two concepts are that the numerator and denumerator in rational numbers must be relatively prime and fractions not to be taken negative values. These inferences are supported by the researches (Erdem et al., 2015; Heritage and Niemi, 2006; Pinto and Tall, 1996) for fractions and others (Balcı, 2006; Behr et al., 1992, Erdem et al., 2015; Hilbert, 2011; King, 2010; Lamon, 2007; Ni, 2001; Papp, 1994; Vamvakoussi and Vosniadou, 2010) for rational numbers.

It is believed that these results will contribute to determining the deficiencies in mathematics teacher education programs and contribute to the professional development processes of the teachers by taking the necessary precautions to eliminate these deficits. Indeed, the importance of the education received by candidate mathematics teachers at the university in effectively teaching in their professional life was previously expressed. In this context, it must be ensured that candidate teachers learn mathematical concepts by associating them with questions as "What is a fraction?", "What is a rational number?", "What kind of a relationship is there between the fraction and rational number?" during university years. In these years, it can be ensured that PMTs teach more effectively in their professional lives by emphasizing the concept errors they may fall into with regard to the concepts fraction and rational number and how can these be remedied.

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