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Analyzing and Comparing Basel III Sensitivity Based Approach for the Interest Rate Risk in the Trading Book

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Abstract

A bank's capital charge computation is a widely discussed topic with new approaches emerging continuously. Each bank computes this figure using internal methodologies in order to reflect its capital adequacy; however, a more homogeneous model is recommended by the Basel committee to enable judging the situation of these financial institutions and relating different banks among each other.

In this paper, we compare different numerical and econometric models to the Sensitivity Based Approach (SBA) implemented by the Basel Committee on Banking Supervision (BCBS) under Basel III in its December 2014 (rev. March 2015) publication in order to compute the capital charge in the trading book. We study the influence of having several currencies and maturities within the portfolio and try to define the time horizon and confidence level implied by Basel's III approach through an application on bonds portfolios.

By implementing several approaches, we are able to find equivalent VaRs to the one computed by the SBA on a pre-defined confidence level (97.5%). However, the time horizon differs according to the chosen methodology and ranges from 1 month up to 1 year.

Keywords: Capital charge, Sensitivity Based approach, Basel III, bonds portfolio, trading book, interest rate risk

1. Introduction

Commercial banks are a key component of today's financial and economic system. Banks allocate funds from depositors to borrowers, convert maturities, and provide financial products. These services among others enhances the efficiency of the overall economy. Given this crucial role, adequate regulations should apply to monitor banks risks.

Since its first issuance in 1988, Basel has been the main banking regulation authority initializing with some main credit risk rules. In 1996, the market amendment was issued setting the basic standards regarding trading assets and the value at risk computation methodology. It also divided the risk into market and credit risk; market risk being divided between equity, interest rate, foreign exchange, commodity, and option risk with a standardized capital computation approach treating each asset class separately. In 2006, Basel II came along with more 'personalized' approaches such as Internal Rating Based (IRB) for credit risk, internal models for Over The Counter (OTC) derivatives exposure along with the introduction of the operational risk charge.

However these regulations did not prevent major crisis hitting the international market causing huge losses in different sectors (cf. Baptistab et al. (2012)). As a response to this shortage, Basel 2.5 was created beginning 2011 as a response adding more capital to the trading book (especially on the poorly modeled products). Basel 2.5 additions target the stressed VaR concept, the incremental risk charge and few new standard rules regarding the banking book.

In 2013, after a thorough observation of the consequences of the crisis and the attempt for correction made by 2.5, Basel III was introduced. The main functions of this issuance are: increasing capital for counterpart exposures, tightening the definition of the bank's capital, adding buffer for liquidity and introducing a new leverage ratio (cf. BCBS (2014)). In parallel, increasing the trading book capital requirement under Basel 2.5 was required following the crisis and was not well designed: the calculation remains non-risk sensitive and highly conservative, and differences in model approval persist between jurisdictions.

As a result, in May 2012 the Basel Committee published a Consultative Paper on a ’Fundamental Review of the Trading Book’ (FRTB) to improve this framework, then in December 2014, "Fundamental review of the trading book: outstanding issues", and its comments March 2015,the BCBS exposed the weaknesses of Basel’s previous approaches. It suggested some major changes to the trading book to be implemented by 2018: scope and approval process (boundary of the trading book, desk level model approval, model testing, model independent assessment tool), modeling Issues (stressed expected shortfall, liquidity adjustments, diversification, default and migration risk, and non-modelable risk factors), and new standard rules for capital charges computation.

The standard rule suggested by the FRTB is based on sensitivities, hence it is called the "Sensitivity based approach" (SBA): it was implemented as the ’homogeneous’ method for capital charge computation across all banks. The SBA is based on percentages and correlations between different maturities and currencies (cf. BCBS (2015)). The existing standard rules poorly reflected hedging or diversification thus inflating the trading book capital level. The SBA is simple yet risk sensitive which is already a big improvement. SBA is a standardized method that reflects the risk resulting from: Interest rate, credit spread, equity, commodity, foreign exchange, options risk and default.

Comparing regulatory approaches, an obvious contrast between Basel and Solvency is noted: Solvency II has a similar three pillar structure as Basel’s Accords. The capital requirements are described under the first pillar and refer to all types of risks: an insurance is exposed to: market risk (interest rate risks, equity risk, property risk, spread risk, concentration risk and currency risk) and counterpart default risk. Both frameworks take diversification effects into account and use square root formulas. However, these aggregation approaches are applied at different levels: a considerably stronger risk differentiation is shown under Basel III. For example, the SBA equity risk distinguishes 10 risk categories in order to assign the risk weights, in contrast to one single shock for all listed equities under Solvency II. Under the interest rate risk, SBA is Basel’s III approach whereas in solvency II a shocked scenarios based computation sets the capital charge (cf. LAAS and Siegel (2015)).

Seemingly, the SBA has several add-ons regarding sensitivity and diversifying considerations however it still has some issues (specifying the coefficients) and details concerning the aggregations that need to be tweaked; for instance the figure under the square root is sometimes negative (cf. ISDA (2015)). In this paper, we aim to focus on the suggested standard rule by the FRTB in order to compare it with other econometric models to find an equivalent capital charge computation technique with few additional details such as time horizon and confidence level for a given capital. Among the different risk modulations, we chose to focus on the interpretation of the interest rate risk capital charge calculation.

According to Oxford Dictionary of Economics, interest rate is defined as "The charge made for the loan of financial capital expressed as a proportion of the loan". More formally, Basel Committee on Banking Supervision (cf. BCBS (2004)) indicated that interest rate risk (IRR) is the exposure of a bank’s financial condition to adverse movements in interest rate.

The sound IRR management conducted by Basel Committee on Banking Supervision had been the source on which analysts rely to evaluate the activities of bank’s risk management of interest rate (cf. BCBS (2004)). In the guideline, the committee offers four basic elements of IRR: appropriate board oversight, comprehensive internal controls, adequate policies and appropriate risk measure. The SBA falls under this forth element in modeling the interest rate risk in the trading book.

Our aim in this paper is to understand the computation of the interest rate risk in banks based on BASEL’s III approach. However, the SBA remains relatively vague and the choices of its coefficient and correlation parameters are not robustly detailed and documented. Hence, studying the interest rate risk from an econometric point of view in order to compare and contrast the results is critical and highly important.

Based on different central banks approaches for term structure interest rate, we selected few econometric models. The main idea was to reduce the dimensions of the database, study the dynamics of these ’reduced’ factors and then conclude on the wider data range dynamic. We introduce the methods used to derive capital requirement and compare them with SBA’s in order to conclude on some equivalence between them.

Having this objective in mind, the structure of the work is as follows: We started by modeling each interest rate curve on a stand-alone maturity basis using a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) approach. It is worth noting that by doing so, we dropped a crucial information which is the strong correlation between these maturities, however we needed this phase as a starting point and a comparison threshold. Modeling the volatility of term structures using GARCH processes has become a current practice due to its numerous advantages relative to alternative models. GARCH-methods are a way of investigating how a function of past returns, in a specific financial series, should be constructed and mapped onto the second moment (cf. Hull (2000)). Proposed by Engle (1982) and then generalized by Bollerslev (1986), GARCH models explain high frequency financial data series through the autoregressive conditional heteroskedasticity and can model simultaneously conditional mean and conditional variance (cf. Edison and

Liang, 1999). Two parameters for orders could be used in order to optimize the results of the tests regarding GARCH coefficients convergence. However, in practice, low orders are more frequently used. The first-order (p=q=1) GARCH model (cf. Taylor (1986)) has become the most popular GARCH model.

Secondly, we introduced the component approaches starting with the Principal component analysis (PCA) which is one of the multivariate analysis techniques usually used for correlation studies, data reduction and efficiency assessment (cf. Levieuge et al. (2010)). This method incorporates the interdependence between term structures maturities: it considers the correlated curves and generates new non-correlated variables. Each factor is related to a loading and a cumulative variance defining the variance explained by each one of the new variables. PCA creates the same number of term structures included in the model however, we need to choose the reduced number of factors that we want to handle. In this work, we chose to cover 98% of the variance, by considering two or three factors. Using a GARCH model, we only project the chosen factors and not the loadings; then we re-create the entire data from the projected factors and previously observed loadings.

Thirdly, we introduced the implementation of the Independent component analysis (ICA): it provides a mechanism of decomposing a given signal into statistically independent components. PCA uses only second order statistical information however, ICA uses higher order (kurtosis) for separating the signals which permits more conclusive results in financial data (cf. Comon (1994)). A drawback in the ICA is its inability to indicate the data variance coverage for each factor, therefore the modeler has to define the number of factors to be considered; in this paper we chose to include three ICA factors.

Regarding the last approach, a factor model is suggested: the Dynamic Nelson Siegel. No GARCH processes are used, instead a mix of Nelson Siegel estimation and Autoregressive Integrated Moving Average (ARIMA) processes projection are put in practice. Yield curve factor models, such as Nelson-Siegel (1988), its dynamic version (cf. Diebold and Li (2006)) and its arbitrage-free counterpart proposed by Christensen, Diebold and Rudebusch (2011), have been extensively applied to forecast bond yields. We used the Dynamic Nelson Siegel due to its flexibility in representation especially for the long term projection. By fitting the curves, projecting the factors using Diebold method and the loadings employing an ARIMA process, we are able to reconstruct the curves from which we concluded the capital requirement.

The capital charge using the previously mentioned methods, except SBA, can be computed on a certain confidence level basis and for a given time horizon; therefore comparing these methods to SBA would determine a common time horizon and confidence level, reaching the purpose of this paper. We start by explaining in details the procedure of the SBA, the correlation between the duration of the portfolios and the capital charge required by this procedure then compare the methods using different approaches. These latter will be based on bonds portfolios denoted in: euros, dollars and Turkish lira from the French, German, US and Turkish governments yields respectively, for maturities between 1 month and 30 years.

In this paper we proceed as follows: in Section 2 we provide a detailed description of the sensitivity based approach through hypothetical portfolios, we also show the link between this capital charge computation and the portfolios duration. In section 3, we proceed with an overview of four different approaches then we explain how we computed the capital charge using these processes. In section 4 we present the empirical analysis, describe the data, estimate the models, compute the VaRs and conclude with analyzing, comparing the capital requirement calculations. In section 5 we offer some interpretation and conclusive remarks.

2. Sensitivity Based approach (SBA)

2.1 Introducing the approach

This new method would require banks to use prices and rate sensitivities in order to compute their capital charge. This revised (sensitivity-based) standardized approach would capture more granular or complex risk factors across different asset classes in the trading book (cf. BCBS (2015)). It builds on the standardized framework tested in the trading book QIS conducted in the second half of 2014 (cf. Basel (2014)).

The proposed methodology covers the delta and optionality risk: general interest rate risk, credit spread risk of non-securitization and securitization exposures, equity, FX risk and commodity. Vega and curvature risk measurements are under development in order to measure the sensitivity of the value of an option with respect to a modification in volatility and the rate of change of delta.

2.2 Implementation reasons

- The approach must provide a method for calculating capital requirements for banks with a level of trading activity that does not require sophisticated measurement of market risk.

- It provides a fall back in the event that a bank's internal model is deemed inadequate, including the potential use as add-on or floor to an internal model-based charge.
- The approach should facilitate consistent and comparable reporting of market risk across banks and jurisdictions.

2.3 Compare and contrast Value at Risk and Expected Shortfall

Since 1996, the Basel Committee has proposed to use the Value-at-Risk (VaR) as an easy to grasp extreme event measure with a certain confidence level. However, better measures of risk are desired for an extra robust risk management such as the expected shortfall.

While Basel II was considering a VaR approach, Basel 2.5 and 3 rejected this method due to several arguments dismissing it as inaccurate in favor of the tailed VaR or, as more commonly known, the expected shortfall. In practice many evidences could question both methods and many evidences could support each.

Comparing these two risk measures we note the following:

- The Value at risk can be misleading: it is seen as 'the maximum loss' therefore it is giving a false sense of security, whilst it is real signification is the threshold of losses in the chosen confidence level meaning that the loss will exceed this VaR, without noting the amount of this excess, the ES covers this shortage.
- ES has better theoretical properties than VaR. If two portfolios are combined, the total ES usually decreases -reflecting the benefits of diversification- and certainly never increases. By contrast, the total VaR can and in practice occasionally does increase: VaR is said to be not coherent because it does not have this particular property.
- The expected shortfall has its shortcomings against the VaR too: First, it is difficult to back-test. A key point is that back-testing a stressed model, whether VaR or ES, is not possible because we are interested in whether the model performs well for another stressed period, but we do not have another such period to use for testing.
- Another disadvantage of ES is that estimates of the measure may not be as accurate as estimates of VaR. The accuracy of VaR and ES is about the same when the loss is normally distributed, but that VaR estimates are more accurate than ES estimates when the losses have fat tails. This means capital calculated from ES may be less stable than capital calculated from VaR.

The knowledge of relationship between different risk measures is important for selecting appropriate risk control strategies.

- On the first hand, for a given risk level p, the ES can be derived by multiplying the VaR with an amplifying factor. In their paper Sensitivity Analysis of Distortion Risk Measures, Gourieroux and Liu (2006), show that this amplifying factor is mostly a function of p and only independent of p if the underlying distribution is Pareto.
- On the other hand, this same publication, proves that the VaR and Tail-VaR can be related through their risk levels by some transformation that could be linear or not.

For parametric distributions several tests and modeling can be made in order to contrast and compare ES and VaR and find the link between these two measures however, in other 'real' situations and dynamic portfolios such approaches are not that easy due to the difficulty of estimating the ES. In this work, we compare the Basel III approach to our models: Basel III leans on an expected shortfall approach whereas we use the value at risk tool. Equivalences will show the convergence between these methods on differing confidence level and time horizons.

2.4 Computational Steps

We first compute the net sensitivity of the bond (relative 1 bps change) and multiply it by its corresponding risk weight in order to get the weighted sensitivity. We note that for each maturity a different risk weight is allocated based on a matrix provided by the Basel committee. For each currency, the 'average' is computed as the square root of the sum of squared single weighted sensitivities and double products of these latter weighted by given, maturity based, correlation coefficients. Aggregation on a portfolio level is another sum of the squared capital charges computed for each currency plus the double products weighted by a factor of 0.5 fixed by Basel. The method is as follows:

1. Get the observed yield and price on the market.

2. Compute the net sensitivity of each instrument and recalculate the price.

3. Based on the matrix imposed by Basel, get the weighted sensitivities (WS), i and j refer to the maturity.

4. Form buckets by sorting each currency in a separate bucket.

5. For each bucket compute the following:

$$K_{bucket} = \sqrt{\sum_{i=1}^{N} WS_i^2 + \sum_{i=1}^{N} \sum_{j \neq i} \rho_{ij} WS_i WS_j} \quad (1)$$

6. Compute the capital charge (having M buckets):

$$Capital\ Charge = \sqrt{\sum_{l=1}^{M} K_l^2 + 0.5\sum_{l=1}^{M}\sum_{p\neq l} S_p \times S_l}\ where\ S_l = \sum_{d=1}^{l} WS_d \text{ for all maturities in bucket l} \quad (2)$$

2.5 Hypothetical example

Let us consider a hypothetical portfolio composed of only one zero coupon bond. The portfolio is studied on a rolling basis and the bond does not include optionality. In this paper, we do not consider the effect of currency or default risk.

Table 1: One zero coupon SBA capital charge

Price (P)	$P = 100\exp(-r_T T)$
Modified price (P')	$P' = 100\exp(-r_T T) - 0.0001T)$
Sensitivity (S)	$S = \frac{P' - P}{0.0001}$
Weighed sensitivity (WS)	$WS = RW \times S$
Capital Charge (CC)	$CC = \frac{WS}{P}$

The previous example proves that the capital charge under the SBA is only dependent of the zero bond maturity (therefore duration) and the associated risk weight of this particular maturity.

Hereafter we consider a portfolio combining three bonds: two in a same currency and a third in a different one.

Let α_1 denote the first currency and α_2 the second one, P_1,P_2 the price of the two bonds in α_1 and P_3 the bond in α_2; τ_1, τ_2 *and* τ_3 their respective maturities.

- For $i \in \{1,2,3\}$ *and* $t_i \in \{0, \dots, \tau_i\}$,the prices and durations are shown as follows:

  $P_i = \sum C_{it} e^{-r_{it}t_i}$ *and* $D_i = \frac{\sum t C_{it} e^{-r_{it}t}}{P_i}$ with C_{it}being the cash flows and r_{it} the interest rate for bond i at time t.

- We compute the net sensitivity NS_i and the weighted sensitivity WS_i:

  $$NS_i = \frac{\sum_t C_{it} e^{-r_{it}t} - \sum_t C_{it} e^{(-r_{it} - 0.01\%)t}}{0.0001}\ and\ WS_i = \frac{RW_i}{0.0001}\sum_t C_{it} e^{-r_{it}t}\left(1 - e^{-0.0001t}\right) = RW_i \times D_i \times P_i$$

- Having two different currencies, two buckets are created and a K_{α_i} is computed for each:

  $K_{\alpha_1} = \sqrt{WS_1^2 + WS_2^2 + 2\rho_{1,2} WS_1 WS_2}$, in the second bucket having only one bond $K_{\alpha_2} = WS_3$

- Bringing these together, SBA demands a 0.5 coefficient for the correlation and a sum of square to compute the capital charge:

$$CC = \frac{\sum_{i=1}^{3} RW_i^2 P_i^2 D_i^2}{\sum_{i=1}^{3} P_i}\left(1 + \frac{2\rho_{1,2} RW_1 D_1 P_1 RW_2 D_2 P_2 + RW_1 D_1 P_1 RW_3 D_3 P_3 + RW_2 D_2 P_2 RW_3 D_3 P_3}{2\sum_{i=1}^{3} RW_i^2 D_i^2 P_i^2}\right) \quad (3)$$

3. Equivalent interest rate risk assessment methods

Computing the capital charge of a given portfolio needs the computation of the value at risk using the traditional approaches. Therefore, we will compute the value at risk combining different known methods in order to compare them with the SBA results.

3.1 ARCH-GARCH

ARCH methods were introduced by Engle in 1982 (cf. Engle (1982)) then generalized by Bollerslev in 1986 (cf. Bollerslev and Tim (2008)) as a conditional variance prediction model, especially useful when the volatility of the financial data is the main issue.

Let X_t denote a real-time stochastic process and σ_t its conditional variance; GARCH (p, q) process is given by:
$X_t | \sigma_t \sim N(0, \sigma|t^2)$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{4}$$

Where $p \geq 0, q \geq 0, \alpha_0 \geq 0, \alpha_i \geq 0 \text{ and } \beta_j \geq 0 \; \forall i, j$

The GARCH approach has been used in modeling financial time series, test financial theories and interpret key features of a given data in a time-dependent matter.

In our bonds portfolio we consider government yield curves with maturities ranging from 3 months to 30 years, compute the return and apply GARCH models for each maturity.

The orders of a GARCH process play a major role in determining the results: q can be based on model selection tests such as the autocorrelation function of the squared residuals; however with large q, estimation error might increase. Finding both p and q parameters can be facilitated through time series testing.

In our paper GARCH models are determined on the basis of coefficients significance and Jarque Berra test (cf. Bollerslev (2008)): skewness and kurtosis are used for constructing Jarque Berra's test statistic to find whether the coefficients of skewness and excess kurtosis are jointly null (cf. Jarque and Bera (1981)).

When estimating GARCH models, computer-based softwares have to be used. Different softwares have different functionalities, drawbacks and features. Several works present these GARCH estimating software packages and compare them (cf. Brooks et al. (2001)). In our work, the 'tseries' R-package is used to estimate these models.

We start by estimating the GARCH process for each maturity, after having adequately selected the parameters. Since the projection requires initial values for each yield curve, we adopted the traditional way of having the last observed yield as the first data point of projection and the volatility as the historically observed volatility for each curve. We project for one year, fixing a year as 252 days, using Monte Carlo simulations then extract the value at risk for different confidence levels from 95% to 99.9 %. The capital charge percentage was computed on a mean relative figure basis, i.e. the mean of all simulations is extracted of the VaR and divided again by the mean (10,000 simulations were in order).

3.2 PCA-GARCH

Principal component analysis is a process used to reduce the dimension of the data (cf. Jackson (1991)): This is useful in extracting a visual representation i.e. by reducing the considered dimensions to a much more compact ones enabling the researcher to represent visually the points. PCA transforms a number of starting points to a much reduced one using optimization criteria. Used criteria might be, among others, minimization of the mean-square error in data compression or finding mutually orthogonal directions in the data explaining a maximal variance.

To apply the principal component analysis, p vectors representing the weight or loadings will be considered such as: $w_k = (w_1, \dots, w_p)_k$. These latters map each row vector x_i of X to a new vector of principal component scores $t_i = (t_1, \dots, t_p)_i$, given by $t_{k_i} = x_i . w_k$.

Therefore instead of having 15 yield curves with different maturities, using the PCA process we reduce it to a number of principle factors that represent more than 98% of our data.

Having these factors (in most cases 2 or 3) we project them on a one year basis using GARCH models and choose their parameters based on the previously mentioned details. After rebuilding the fifteen maturities using the projected factors and the previously computed loadings, we generate the VaR and capital charge of our portfolios using the same way selected in the previous methodology for the capital charge computation.

This process adds the correlation between different maturities and shows the interdependency between all tenors even when the portfolio does not include the entirety of maturities.

3.3 ICA-GARCH

Independent components analysis, (cf. Comon (1994)), is another method to reduce dimensions that has the same functionality as PCA except for the difference in the determination of the components and the loadings: In PCA, the aim is to find vectors that best explain the variance of the data whereas in ICA the kurtosis is in focus. The latent variables are assumed non-Gaussian and mutually independent.

ICA could be used in different fields such as digital imaging, stocks databases, economic indicators, geologic measurements or even psychometric indicators. Initially, the process was mostly used to 'un-mix' several signals: different waves recorded at the same time, two time series interfering for a certain process, underwater signals…

In this ICA method follows the same process as PCA's: ICA to the full data panel, GARCH projection, rebuilding of the data, determining the VaR, capital requirement computation.

Again this model includes correlation as well as GARCH estimations however it is an add-on to the previous method due to the following: ICA does not assume the non-correlation of the factors instead it supposes the independence, such that the normality of the data is not a must; on the contrary, non-Gaussian factors have an added-value.

PCA and ICA differences are largely discussed in the literature (cf. Bugli (2007), Burgos (2013)): Both methods are given multivariate measurements with the purpose of dimensions reduction: finding the most fitted smaller representative space therefore no redundancy. However, in PCA the tool to do so is by measuring the intercorrelation between data while in ICA independence is used and the number of 'reduced' variables is less important (cf. Hyvarinen et al. (2001)).

ICA algorithm implementations started with Herault method Herault and Jutten (1986). Since, various approaches followed, suggesting each time a new methodology: minimizing higher order moments (cf. Cardoso (1989)) or higher order cumulates (cf. Cardoso and Souloumiac (1993)), minimization of mutual information of the outputs or maximization of the output entropy (cf. Bell and Sejnowski (1995)), minimization of the Kullback-Leibler divergence between the joint and the product of the marginal distributions of the outputs (cf. Amari et al. (1996)).

This work is applied using fastICA algorithm implemented in a R-package (cf. Delac et al. (2006)).

3.4 Dynamic Nelson Siegel

The interest rate curve is essential for pricing, hedging and evaluating a portfolio. Various curve fitting spline methods have been introduced such as quadratic and cubic splines (McCulloch (1971, 1975)), exponential splines (Vasicek and Fong (1982)), B splines (Shea (1984))... However, these methods were criticized for not being too representative of the economic situations. Therefore, Nelson and Siegel (1987) and Svensson (1994, 1996) suggested parametric curves that are flexible enough to describe the large frame of the financial conditions.

Nelson Siegel method consists of estimating three parameters using the maximum likelihood process or OLS to rebuild the yield curve (cf. Siegel and Nelson (1988)): the three Nelson-Siegel components have a clear interpretation as short, medium and long-term components. These labels are the result of each element's contribution to the yield curve.

$$y(\tau) = \beta_1 + \beta_2 \frac{1-e^{-\lambda\tau}}{\lambda\tau} + \beta_3 \frac{1-e^{-\lambda\tau}}{\lambda\tau - e^{-\lambda\tau}} \tag{5}$$

Where $\beta_1 > 0, \beta_1 + \beta_2 > 0 \; and \; \lambda > 0$

The Nelson Siegel model is extensively used by central banks and monetary policy makers (ex: Bank of International Settlements (2005), European Central Bank (2008)).

As a development to the traditional fitting approach, Diebold and Li (2006) introduce the Dynamic Nelson-Siegel (DNS) model by estimating the classical formula with time-varying factors and model them using autoregressive specifications projecting therefore the yield curves by adding dynamism to the parameters. Diebold and Li (2006) interpret $\beta_{1t}, \beta_{2t} \; and \; \beta_{3t}$ as the slope, curvature and level of the curve. This method shows very encouraging results especially on a long time horizon.

$$y_t(\tau) = \beta_{1t} + \beta_{2t} \frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} + \beta_{3t} \frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau - e^{-\lambda_t\tau}} \tag{6}$$

Our chosen approach in this paper is to fit the yield curves using the traditional Nelson Siegel and to project it afterwards: estimating the different yields using the Nelson Siegel function in R, R-package: YieldCurve, projecting the betas computed using adequate ARIMA processes (based on best-fit approach); maintaining the loadings as calculated historically, rebuilding the projected yields based on Diebold and Li's dynamic approach. Having the projected yields, a new portfolio evaluating could be placed, a value at risk and therefore a capital charge is computed.

Lambda: The lambda factor in the Nelson Siegel formula, as provided by Nelson and Siegel (1987), does not have any exact economic meaning. In fact, from the econometric point of view, it is a constant assuring a specific slope of yield curve. In practice, higher values produce a faster decay of yield curve and vice versa. In conclusion, the factor plays a key role especially in fitting the longer end of yield curves.

Nelson and Siegel (1987) fixed λ to simplify the computation allowing usage of the simple and linear ordinary least squares. Diebold and Li (2006) follow the same logic of exogenously predetermined λ however they suggested interpretation for the parameters as the slope, curvature and level. Based on such assumptions, and interpreting lambda as the maturity at which the loading on the curvature factor achieves its maximum, maximizing this loading would result in a numeric value for this coefficient: 0.0609.

Alternatively, in the related literature, Diebold et al. (2006) instead use the value of 0.077. Adding to that, other papers suggest different approaches for computing lambda not related to the aforementioned interpretation (cf. Comisef (2010)) and (cf. Gilli et al. (2012)).

In this paper, lambda of the projected yield is also considered as a constant chosen as the mean of the previously estimated lambdas for each specific yield.

4. Application: Bonds Portfolios

4.1 Data used

The data is fetched from Bloomberg: Government yield curves 3 months up till 30 years maturities on a daily basis for: France, Germany, USA and Turkey. We chose to go with this selection in order to cover a heterogeneous sample having three different market conditions: two European stable markets, the American market and an emerging case such as Turkey. For each country, different dates are available, France data starts on 04/30/1998, German on 10/04/1991, US on 11/24/2003 and Turkey on 04/01/2005; all the data ending point is on 05/15/2015.

4.2 Portfolios' duration and different capital charge requirements

In order to compare these methods, we build portfolios using either one unique currency or multiple currencies. The following plots represent the different yields for each currency:

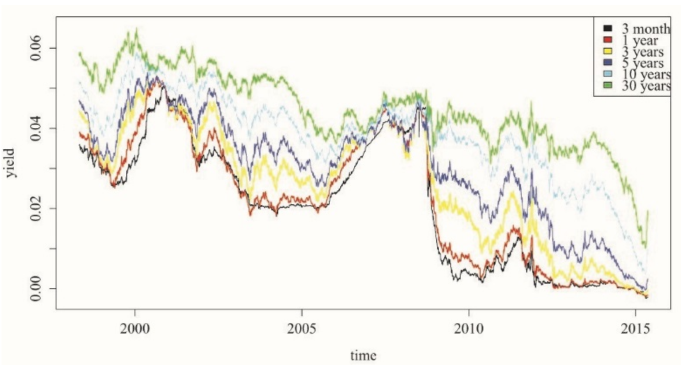

Figure 1. French Government Curves

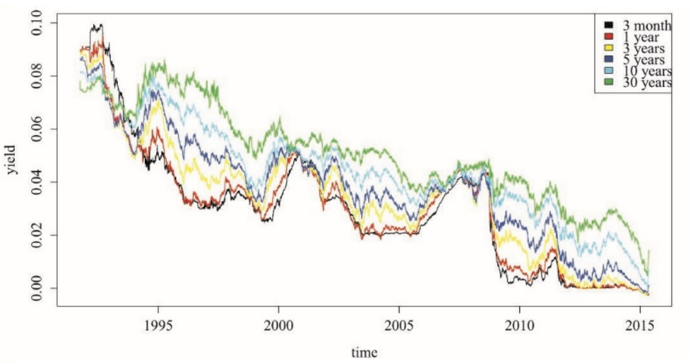

Figure 2. German Government Curves

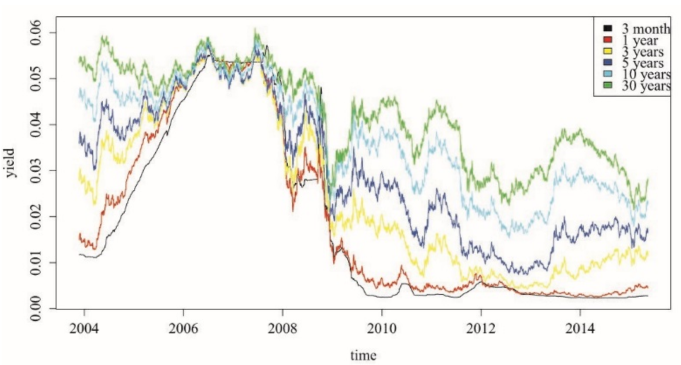

Figure 3. US Government Curves

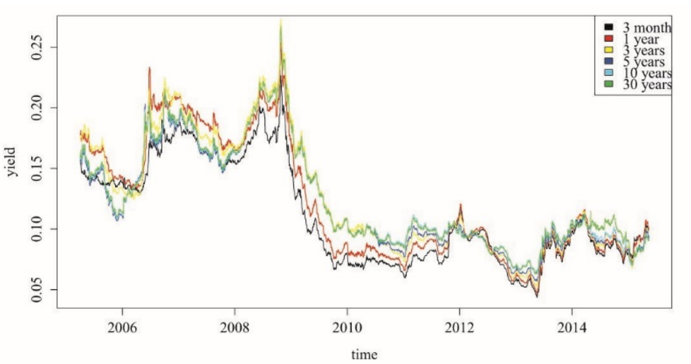

Figure 4. Turkish Government Curves

In these plots we can distinguish four phases:

1. The normal phase up until 2004.
2. The moderation phase as called by Basel between 2004 and 2007, ending with the beginning of the financial crisis (low volatility).
3. The liquidity crisis between 2007 and 2008.
4. The zero bound phase where the volatility decreases going from 2008 up till the end of the sample: low short term volatility, high long term volatility.

4.3. Single currencies portfolios capital charge

We consider four portfolios: for each governmental yield, with the same composition, consisting of 12 zero coupon bonds denoted in the local currency each consisting of: 3 one year ZC, 3 two years ZC, 2 three years ZC, 1 five years ZC, 1 ten years ZC and 2 fifteen years ZC.

4.3.1. Sensitivity Based Approach:

SBA capital requirement is compared below to the portfolio's duration at every date:

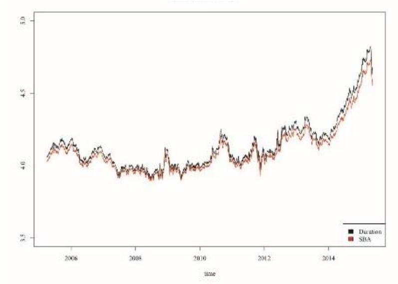

Figure 5. French Duration vs SBA

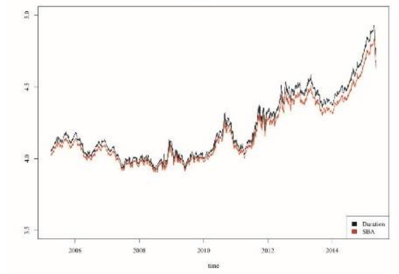

Figure 6. German Duration vs SBA

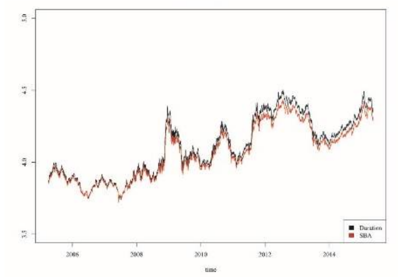

Figure 7. US Duration vs SBA

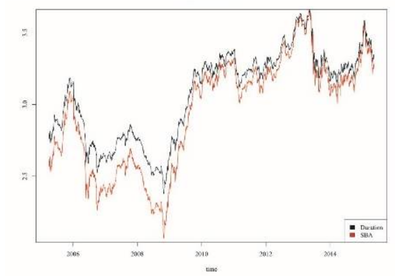

Figure 8. Turkish Duration vs SBA

As previously shown by the three cases equations, the plots reflect the correlation between the duration of the four chosen portfolios and the capital requirement computed using the SBA method.

It is clear that the coefficient depends on a certain factor reflected by the risk weight used. Having the same portfolio composition, the factor between the duration and the capital charge is the same. After noting the obvious resemblance between the behavior of the SBA capital requirement and the duration of these single currency portfolios, we proceed in applying the main goal of this paper.

Our aim is to define an equivalent to the SBA capital requirement using a VaR on a given confidence level and time horizon; we proceed as follows: For each chosen methodology, we compute the VaR of the portfolios price (Monte-Carlo simulations basis) and compare these VaRs to the SBA requirement; the interception between these figures will simulate the confidence level and time horizon equivalent to Basel's requirement. We initiate this comparative work by computing the GARCH process value at risk.

Please note that in the following plots red lines represent the SBA capital charge, black a 99.8 % confidence level, green a 99%, blue the 97.5% and magenta a 95 % confidence level.

4.3.2 GARCH model:

For each currency, we have 15 yield curves with different maturities; we compute: $\Delta i_t = i_t - i_{t-1}$. Building on these differences, we estimate them using an GARCH model; projecting this model one year ahead (252 days) we build our 'future' portfolios. Repeating this process 10000 times, based on a Monte Carlo logic, we conclude the VaR and therefore the capital charge. Note that for all methods the SBA capital charge is computed as a percentage of the initial portfolio, in the other methods capital charge is computed as the relative change between the projected mean and values at risk of the initial portfolio value.

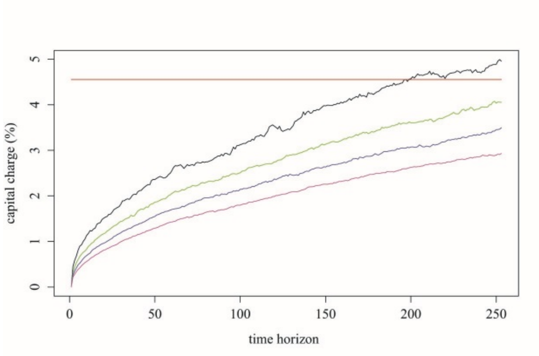

Figure 9. French GARCH capital charge

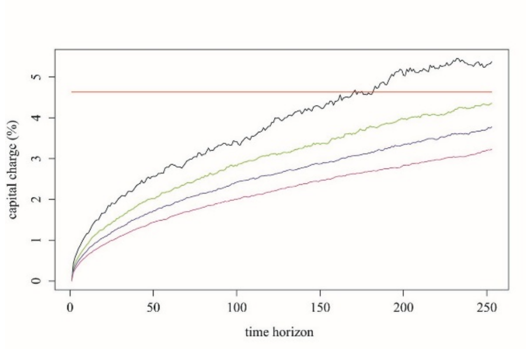

Figure 10. German GARCH capital charge

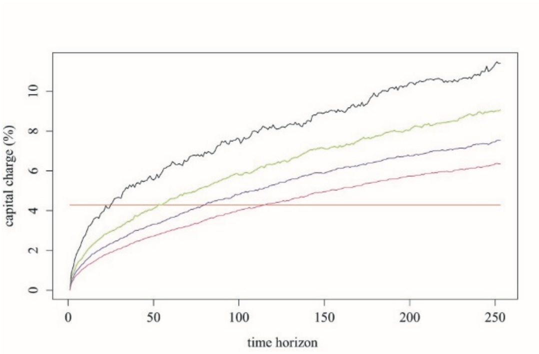

Figure 11. US GARCH capital charge

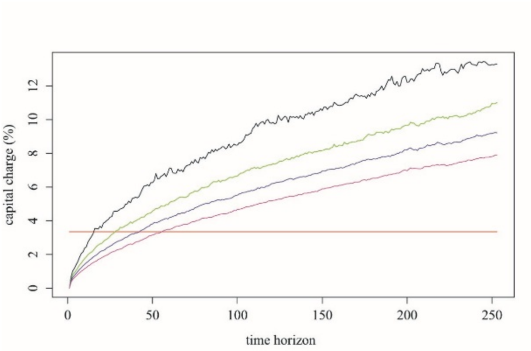

Figure 12. Turkish GARCH capital charge

Not accounting for the inter-correlation in our GARCH approach, the capital charge is expected to fall mostly below the SBA approach. We can observe the resemblance between the French, German and US market, however a very unstable behavior in the Turkish data: Turkey is located in a very fragile environment quickly influenced by numerous factors making its currency very volatile.

4.3.3 PCA-GARCH Model

Applying the PCA on the 15 maturities of each currency, we reduce the data into 2 components covering 98 % of the data. We project the first differential of these components using an adequate GARCH model then rebuild the entire maturities using projected factors and previously computed loadings. Monte Carlo simulations permit the extraction of the VaR at different levels and the computation of the capital charge.

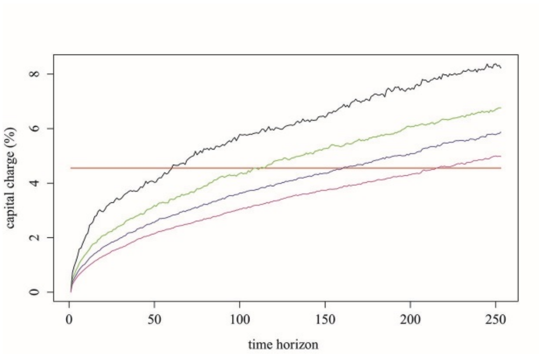

Figure 13. French PCA capital charge

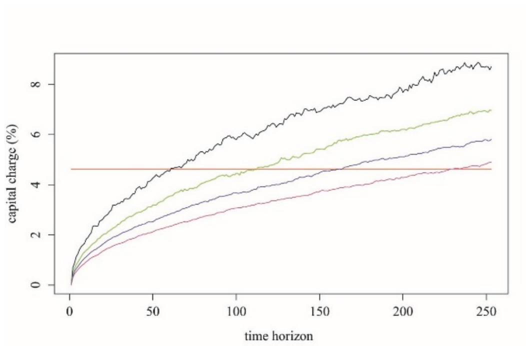

Figure 14. German PCA capital charge

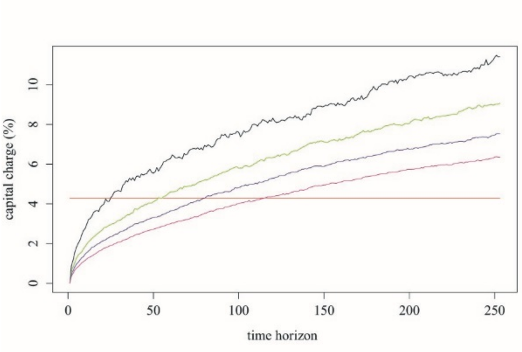

Figure 15. US PCA capital charge

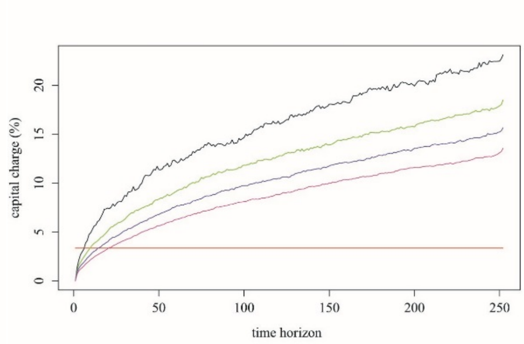

Figure 16. Turkish PCA capital charge

Once again EU and US data show the same behavior, whereas the Turkish data being very volatile shows different results (emerging market status). Quickly comparing GARCH and PCA, a clear increase in the required capital is showing in the PCA figures due to the incorporation of the inter-maturities correlation.

4.3.4 ICA-GARCH Model

This method is similar to the PCA, but instead of using the Principal Components approach we used the Independent Components Analysis to increase the precision and reduce the assumptions.

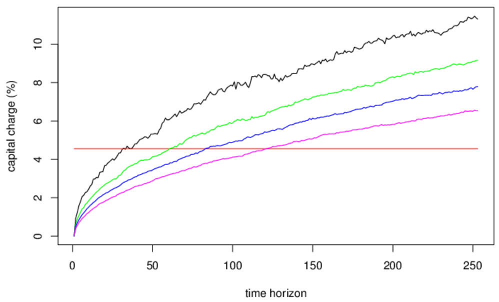

Figure 17. French ICA capital charge

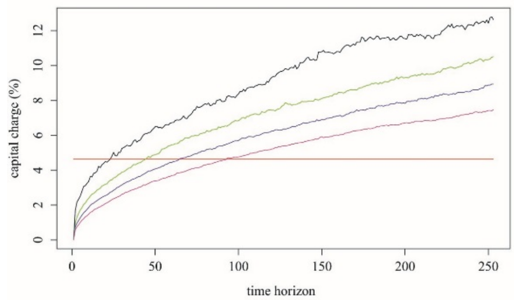

Figure 18. German ICA capital charge

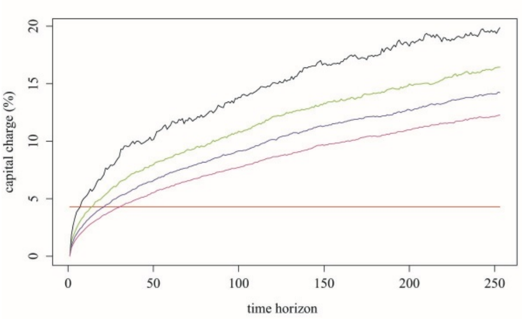

Figure 19. US ICA capital charge

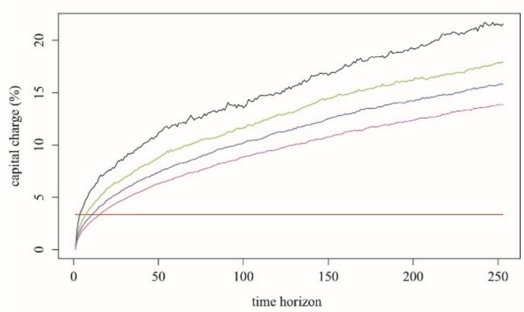

Figure 20. Turkish ICA capital charge

4.3.5 DNS-ARIMA model:

After estimating the curves using NS model, we projected the beta parameters using the best fitted ARIMA(p,d,q) process. Along with the mean of the historically observed lambda's and the projected beta's, we rebuild the curves, estimate the VaR and capital charge.

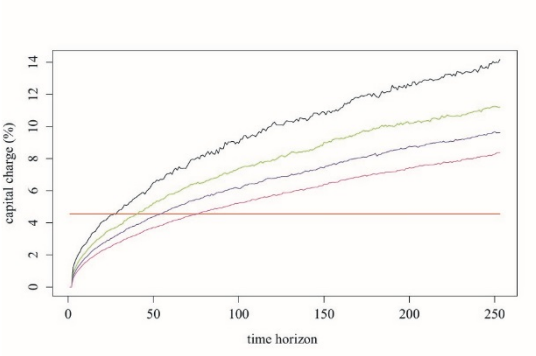

Figure 21. French DNS capital charge

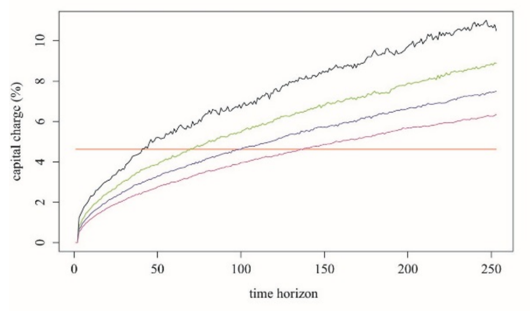

Figure 22. German DNS capital charge

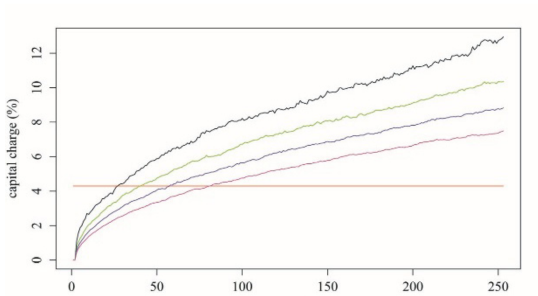

Figure 23. US DNS capital charge

The results show a quicker convergence in this method. However, in an unstable market such as the Turkish case, we do not observe any convergence in the Nelson Siegel parameter, therefore no projection could be applied.

4.4 Multiple currencies portfolios capital charge

Denoting the previous portfolios by their government yield we have: Port_FRANCE, Port_GERMANY, Port_US and Port_TURKEY. In this section we consider multiple currencies combining the previously mentioned portfolios: Port_FR_GR, Port_FR_US, Port_FR_TRY, Port_FR_GR_US, Port_GR_US_TRY and Port_FR_GR_US_TRY

4.4.1 Sensitivity Based Approach

In this section, having multiple currencies, the correlation parameters between different curves at different tenors will be added. Note that both France and Germany are held in euros therefore they are part of the same bucket.

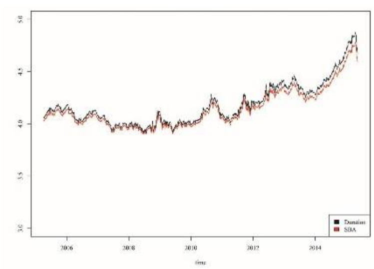

Figure 24. FR-GR duration vs SBA

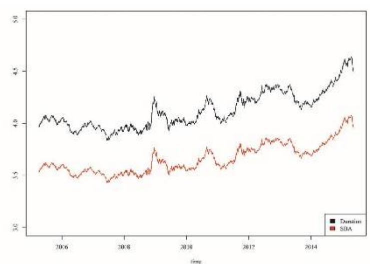

Figure 25. FR-US duration vs SBA

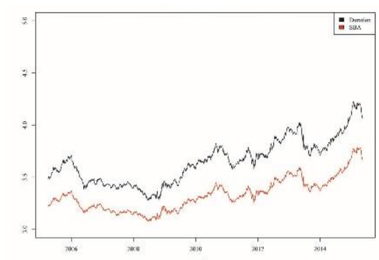

Figure 26.FR-TRY duration vs SBA

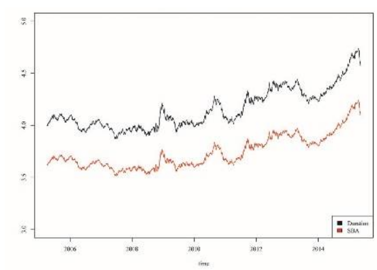

Figure 27. FR-GR-US

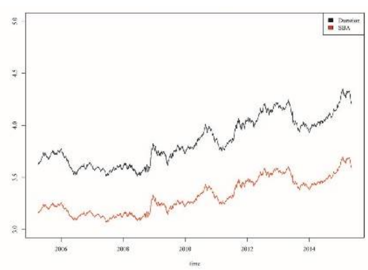

Figure 28. GR-US-TRY

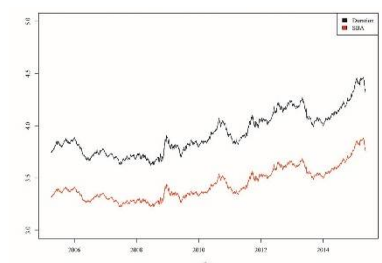

Figure 29. FR-GR-US-TRY

Similar remarks could be presented regarding the duration parallel movement with the capital requirement.

4.4.2 Different portfolios capital charge

FR-GR portfolio

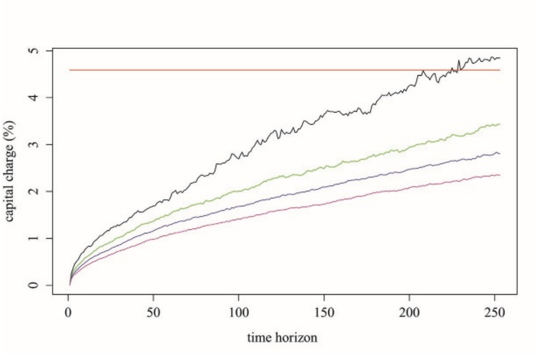

Figure 30: GARCH capital charge

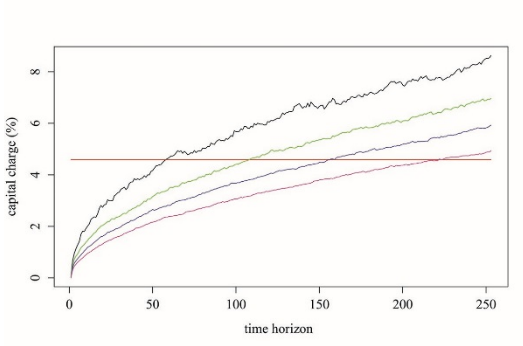

Figure 31. PCA capital charge

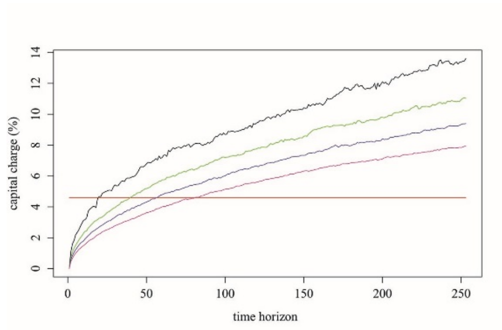

Figure 32. ICA capital charge

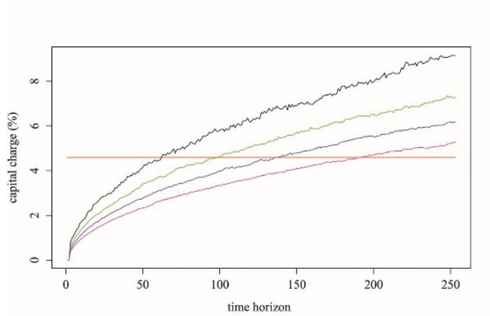

Figure 33. DNS capital charge

FR-US portfolio

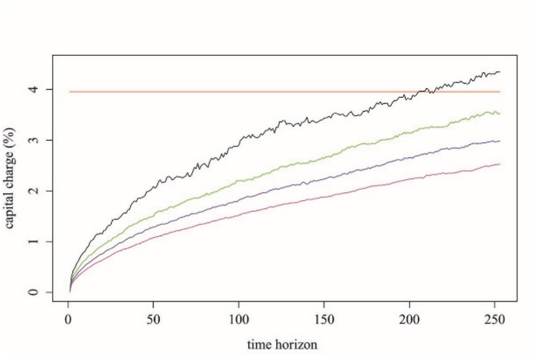

Figure 34. GARCH capital charge

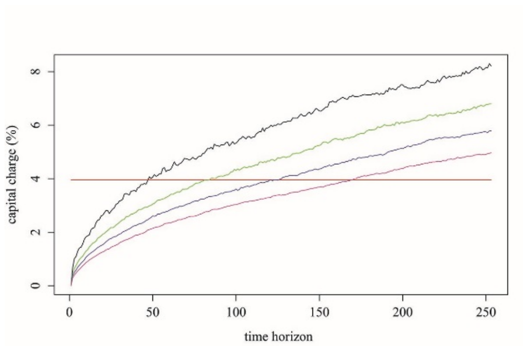

Figure 35. PCA capital charge

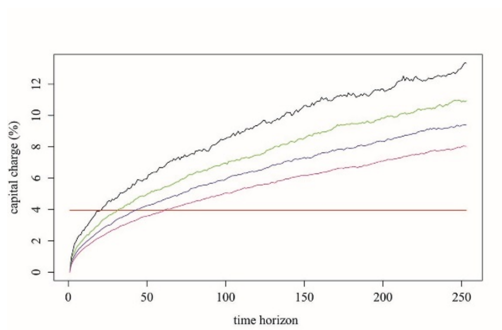

Figure 36. ICA capital charge

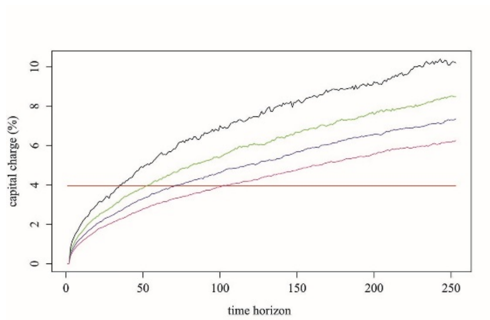

Figure 37. DNS capital charge

FR-TRY portfolio

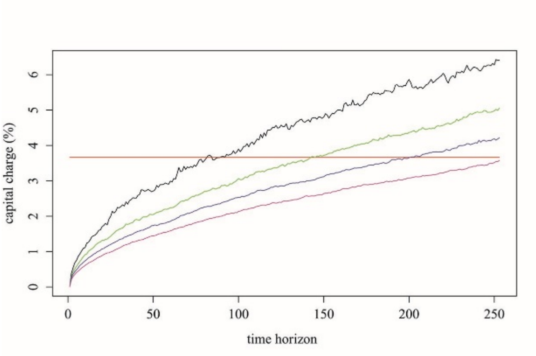

Figure 38. GARCH capital charge

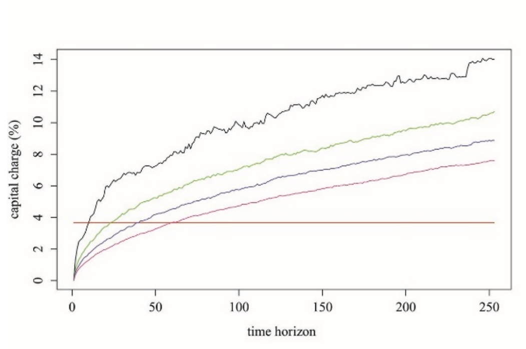

Figure 39. PCA capital charge

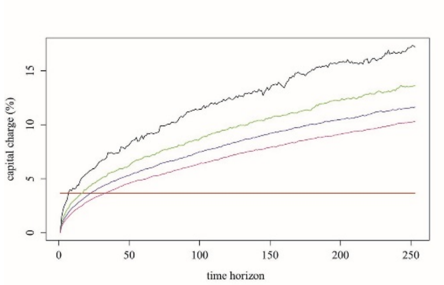

Figure 40. ICA capital charge

FR-GR-US portfolio

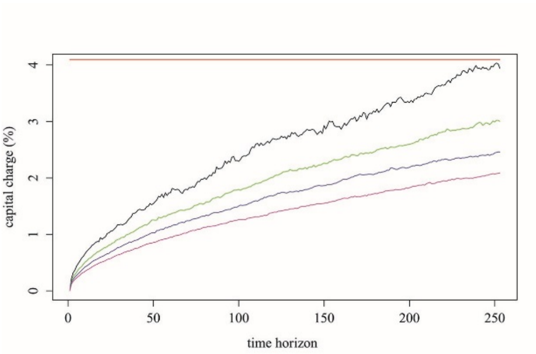

Figure 41. GARCH capital charge

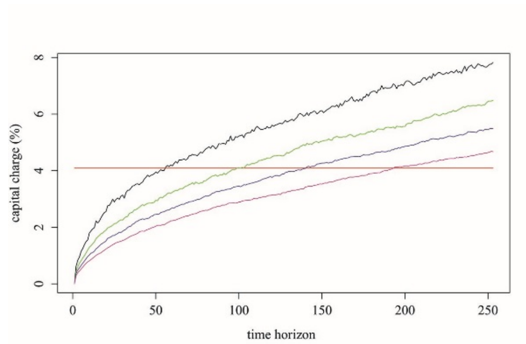

Figure 42. PCA capital charge

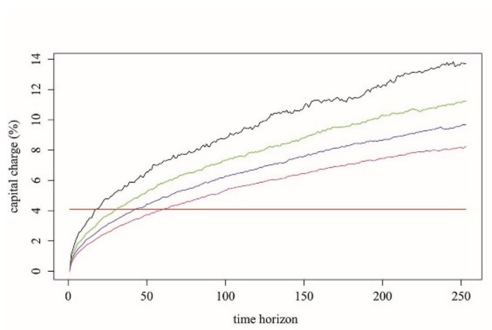

Figure 43. ICA capital charge

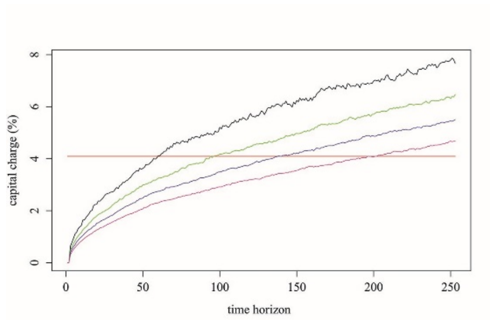

Figure 44. DNS capital charge

GR_US_TRY portfolio

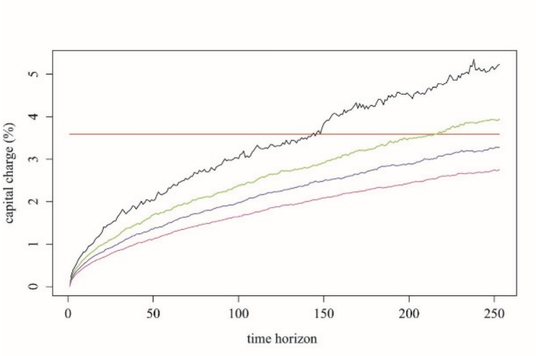

Figure 45. GARCH capital charge

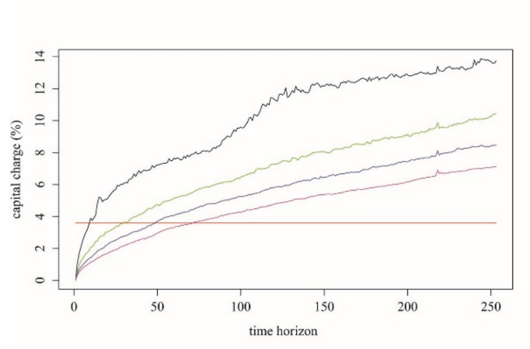

Figure 46. PCA capital charge

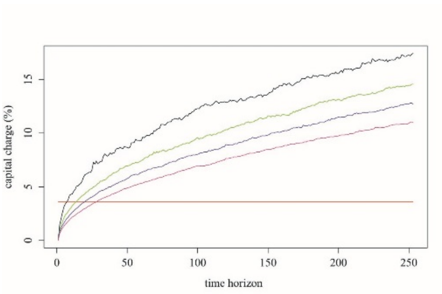

Figure 47. ICA capital charge

FR-GR-US-TRY portfolio

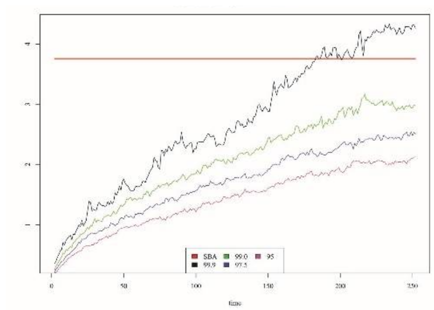

Figure 48. GARCH capital charge

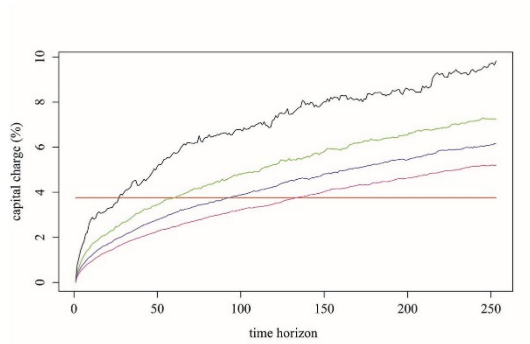

Figure 49. PCA capital charge

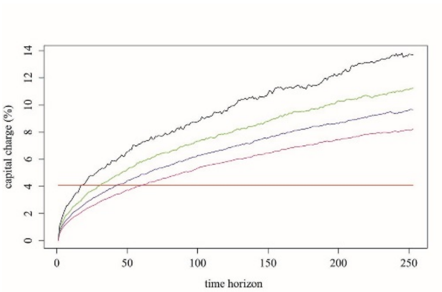

Figure 50. ICA capital charge

4.5 Comparing results

In order to reduce the fluctuation, the results were interpolated to a logarithmic function resulting in the following: Based on the single currency portfolios, GARCH model shows a permanent 'low' level of capital requirement compared to Basel and the other methods. This could be explained by the correlations' exclusion between different tenors in this first method. PCA and ICA methods show a close trend however, the ICA approach is more conservative and demands higher requirements. In all portfolios, DNS converges very rapidly, equaling the SBA on a short time horizon.

5. Conclusion

We have compared in this paper different methods for computing the capital charge of a commercial bank based on a Eurobonds portfolio example and we have explored the performance in an out- of-sample forecasting based on the number of violations. These approaches might be used as internal models compared to Basel's SBA in order to define a chosen time horizon and confidence level vis a vis the 'standard method'.

Table 2. Encounter in days with the SBA requirement

	GARCH			PCA			ICA			DNS		
	95%	**97.5%**	**99%**	**95%**	**97.5%**	**99 %**	**95%**	**97.5%**	**99 %**	**95%**	**97.5%**	**99 %**
FR	>252	>252	>252	210	150	115	112	77	58	66	52	35
GR	>252	>252	>252	223	156	110	90	57	48	140	100	70
US	118	72	54	120	80	52	31	25	12	80	55	47
TRY	55	41	25	24	18	12	15	12	8			
FR_GR	>252	>252	>252	212	152	105	75	51	40	188	146	100
FR_US	>252	>252	>252	175	125	80	62	47	38	100	60	50
FR_TRY	>252	200	142	68	42	25	40	22	16			
FR_GR_US	>252	>252	>252	200	142	112	55	48	36	182	133	98
GR_US_TRY	>252	>252	210	77	50	28	33	24	21			
FR_GR_US_TRY	>252	>252	>252	138	100	62	62	48	25			

This table summarizes the encountering points (in days) between the SBA and the different other methodologies studied for the ten portfolios

Trying to make sense out of these data we conclude the following results:

Table 3. Color coded encounter dates as months

	GARCH	PCA	ICA	DNS	GARCH	PCA	ICA	DNS	GARCH	PCA	ICA	DNS
	95%				**97.50%**				**99.00%**			
FR	12.00	10.00	5.33	3.14	12.00	7.14	3.67	2.48	12.00	5.48	2.76	1.67
GR	12.00	10.62	4.29	6.67	12.00	7.43	2.71	4.76	12.00	5.24	2.29	3.33
US	5.62	5.71	1.48	3.81	3.43	3.81	1.19	2.62	2.57	2.48	0.57	2.24
TRY	2.62	1.14	0.71		1.95	0.86	0.57		1.19	0.57	0.38	
FR_GR	12.00	10.10	3.57	8.95	12.00	7.24	2.43	6.95	12.00	5.00	1.90	4.76
FR_US	12.00	8.33	2.95	4.76	12.00	5.95	2.24	2.86	12.00	3.81	1.81	2.38
FR_TRY	12.00	3.24	1.90		9.52	2.00	1.05		6.76	1.19	0.76	
FR_GR_US	12.00	9.52	2.62	8.67	12.00	6.76	2.29	6.33	12.00	5.33	1.71	4.67
GR_US_TRY	12.00	3.67	1.81		12.00	2.38	1.14		10.00	1.33	1.00	
FR_GR_US_TRY	12.00	6.57	2.95		12.00	4.76	2.29		12.00	2.95	1.19	

Approaching Basel's method, the results show:

- For European markets, GARCH method computes a similar capital requirement as the SBA for a minimum of one year time horizon.
- Comparing PCA and ICA we can conclude that the ICA is more restrictive for single currency denoted and mixed portfolios.

- PCA gives a seven months, 97.5% adequacy parallel to the three months given by the ICA for FR and GR portfolios. We add a PCA requirement of 4 months horizon for the US portfolio facing a 1 month given by the ICA.
- In the mixed portfolios PCA sets (at 97.5%) a limit of six to seven months except when the Turkish lira is involved, ICA shows a lower encounter point of two months.
- The DNS on the 97.5% gives an adequate capital charge between three and five months.
- In the mixed portfolios DNS method on the 97.5% requires a time horizon between three and seven months.

Based on the previous, our recommendations are:

- For the Eurozone:
    - GARCH capital charge at twelve months would be equivalent to the SBA for a level of 97.5 %.
    - GARCH does not account for inter-maturities correlations therefore an ICA or PCA approach would be more rational:
        - 7 months PCA 97.5 % on a country level and when combined.
        - 3 months ICA 97.5 % on a country level and 2 months when combined.
    - DNS would inquire an average of 3 months for each country and 6 months for a multi-European portfolio.
- For the US:
    - GARCH imposes a 4 month horizon for 97.5% confidence level.
    - 97.5% PCA for less than 4 months capital charge would do it and a 1 month ICA.
    - An average of 3 months DNS results in a close capital charge as the SBA's requirement.
- For the Turkish market:
    - TRY is too volatile to be adequately represented by ICA or DNS models: it can be used for very short term: one month or less PCA (97.5%).
- When combining US and Euro markets:
    - GARCH results could remain applicable (at the one year horizon).
    - PCA method time horizons' is half a year.
    - ICA results in a 2 month time horizon.
    - DNS methods horizon is between 3 and 6 months (due to the change in the distribution of the portfolio and the weakness of the VaR in that case).
- When combining the Turkish lira with any of the US dollar or Euro portfolio: PCA and ICA approach an average of 2.5 months.

The goal of this paper was to provide banks with a tool that explains 'econometrically' Basel's SBA approach in order to fix the time horizon and confidence level of their capital requirement in the trading portfolio. In addition, these models could provide an internal approach with customized coefficients and parameters.

In June 2015, a new consultative document was issued by the BCBS on the 'Interest rate risk in the banking book, presenting new approaches to handle this book's capital charge computation and suggests dividing this amount between the first and second pillar. Incorporating the banking book in the first pillar is a new approach, because that segment was reserved for the trading book. Doing so, a similar methodology to the SBA would be inquired for the banking book. Our next step would be to construct an internal model that mimics the proposed approach to compute the local parameters for the capital charge computation and interest rate shock scenarios applications.

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